Solutions

Nov 20-18:35

6. Determine the break-even quantities for each profit function, where x is the number sold, in thousands.

a) $P(x) = -x^2 + 12x + 28$

$0 = -x^2 + 12x + 28$

$0 = x^2 - 12x - 28$

$0 = (x-14)(x+2)$

$x - 14 = 0 \quad x + 2 = 0$

$x = 14 \quad x = -2$

$\Rightarrow$ 14,000 or −2000 items

not real in this context [extraneous]

c) $P(x) = -2x^2 + 22x - 17$

Break-even when profit equals zero

$0 = -2x^2 + 22x - 17$

won't factor

$\Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-22 \pm \sqrt{(-22)^2 - 4(-2)(-17)}}{2(-2)}$

$x = \dfrac{-22 \pm \sqrt{348}}{-4}$

$x = 0.836$ or 10.164

$\Rightarrow$ 836 or 10164 items

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6. Determine the break-even quantities for each profit function, where x is the number sold, in thousands.

b) $P(x) = -2x^2 + 18x - 40$

$0 = -2x^2 + 18x - 40$

$0 = x^2 - 9x + 20$

$0 = (x-4)(x-5)$

$x-4=0 \quad x-5=0$

$x=4 \quad x=5$

$\Rightarrow$ 4000 or 5000 items

d) $P(x) = -0.5x^2 + 6x - 5$

won't factor

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-6 \pm \sqrt{(6)^2-4(-0.5)(-5)}}{2(-0.5)}$

$x = \frac{-6 \pm \sqrt{26}}{-1}$

$x = 0.901$ or 11.099

$\Rightarrow$ 901 or 11099 items

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7. The flight of a ball hit from a tee that is 0.6 m tall can be modelled by the function $h(t) = -4.9t^2 + 6t + 0.6$, where $h(t)$ is the height in metres at time t seconds. How long will it take for the ball to hit the ground?

A

Hits the ground when the height = 0

$0 = -4.9t^2 + 6t + 0.6$

won't factor, so....

$t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$t = \frac{-6 \pm \sqrt{(6)^2-4(-4.9)(0.6)}}{2(-4.9)}$

$t = \frac{-6 \pm \sqrt{47.76}}{-9.8}$

$t = 1.32$ sec or -0.93 sec

extraneous

Lands after 1.32 seconds

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8. The population of a region can be modelled by the function $P(t) = 0.4t^2 + 10t + 50$, where $P(t)$ is the population in thousands and t is the time in years since the year 1995.
a) What was the population in 1995?
b) What will be the population in 2010?
c) In what year will the population be at least 450 000? Explain your answer.

a) Sub in $t = 0$

$P = 0.4(0)^2 + 10(0) + 50$

$P = 50$

$\Rightarrow$ 50,000 people in 1995

b) $t = 2010 - 1995 = 15$

$P = 0.4(15)^2 + 10(15) + 50$

$P = 290$

$\Rightarrow$ 290,000 people in 2010

c) Population of 450,000 $\Rightarrow$ Solve for $P = 450$

$450 = 0.4t^2 + 10t + 50$

$0 = 0.4t^2 + 10t - 400$ (÷ by common factor 0.4)

$0 = t^2 + 25t - 1000$ (won't factor)

$t = \frac{-25 \pm \sqrt{(25)^2 - 4(1)(-1000)}}{2(1)}$

$t = \frac{-25 \pm \sqrt{4625}}{2}$

$t = 21.504$ or -46.504 ← extraneous

Year = 1995 + 21.54

$\simeq$ 2017

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9. A rectangle has an area of 330 m^2. One side is 7 m longer than the other. What are the dimensions of the rectangle?

Area = length × width

$330 = x(x + 7)$

$330 = x^2 + 7x$

$0 = x^2 + 7x - 330$

$x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(-330)}}{2(1)}$

$x = \frac{-7 \pm \sqrt{1369}}{2}$

$x = 15$ or -22 ← extraneous

Dimensions are

length = 15 m

width = 22 m

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10. The sum of the squares of two consecutive integers is 685. What could the integers be? List all possibilities.

Consecutive means one after the other

$\Rightarrow (x)^2 + (x+1)^2 = 685$

$x^2 + (x^2 + x + x + 1) = 685$

$2x^2 + 2x + 1 = 685$

$2x^2 + 2x - 684 = 0$

$x^2 + x - 342 = 0$

$(x-18)(x+19) = 0$

$x = 18$ or -19

$\Rightarrow x+1 = 19$ or -18

Pairs could be 18 and 19 or -19 and -18

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11. A right triangle has a height 8 cm more than twice the length of the base. If the area of the triangle is 96 cm^2, what are the dimensions of the triangle?

$\text{Area} = \frac{\text{base} \times \text{height}}{2}$

$96 = \frac{(x)(2x+8)}{2}$

$192 = x(2x+8)$

$192 = 2x^2 + 8x$

$0 = 2x^2 + 8x - 192$

$0 = x^2 + 4x - 96$

$0 = (x-8)(x+12)$

$x = 8$ or -12 ← extraneous

Dimensions

base = 8 cm

height = 24 cm

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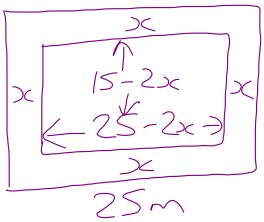
12. Jackie mows a strip of uniform width around her 25 m by 15 m rectangular lawn and leaves a patch of lawn that is 60% of the original area. What is the width of the strip?

T

Area lawn $= 25 \times 15$
$= 375 m^2$

Remaining patch $= 0.6(375)$
$= 225 m^2$



Dimensions of remaining patch are
$15-2x$ and $25-2x$

$\Rightarrow (15-2x)(25-2x) = 225$
$375 - 30x - 50x + 4x^2 = 225$
$4x^2 - 80x + 375 = 225$
$4x^2 - 80x + 150 = 0$
$2x^2 - 40x + 75 = 0$

$x = \frac{40 \pm \sqrt{(-40)^2 - 4(2)(75)}}{2(2)}$

$x = \frac{40 \pm \sqrt{1000}}{4}$

$x = 17.9$ or 2.1
extraneous

width of strip is 2.1m

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13. A small flare is launched off the deck of a ship. The height of the flare above the water is given by the function $h(t) = -4.9t^2 + 92t + 9$, where $h(t)$ is measured in metres and t is time in seconds.

a) When will the flare's height be 150 m?
b) How long will the flare's height be above 150 m?

a) $150 = -4.9t^2 + 92t + 9$
$0 = -4.9t^2 + 92t - 141$

$t = \frac{-92 \pm \sqrt{(92)^2 - 4(-4.9)(-141)}}{2(-4.9)}$

$t = \frac{-92 \pm \sqrt{5700.4}}{-9.8}$

$t = 1.684$ or 17.092 seconds

b) The quadratic opens down so the vertex is between these times.
$\Rightarrow$ length of time above 150m
$= 17.092 - 1.684$
$= 15.408$ seconds

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14. A bus company has 4000 passengers daily, each paying a fare of $2. For each $0.15 increase, the company estimates that it will lose 40 passengers per day. If the company needs to take in $10 450 per day to stay in business, what fare should be charged?

Revenue = # passengers × fare

Let x = # of $0.15 increases

$$10450 = (4000 - 40x)(2 + 0.15x)$$

$$10450 = 8000 + 600x - 80x - 6x^2$$

$$0 = -2450 + 520x - 6x^2$$

$$0 = 6x^2 - 520x + 2450$$

$$x = \frac{520 \pm \sqrt{(-520)^2 - 4(6)(2450)}}{2(6)}$$

$$x = \frac{520 \pm \sqrt{211600}}{12}$$

$x = 81.\overline{6}$ or 5

$\Rightarrow$ New fare

$= 2 + 0.15(5)$

$= \$2.75$

[or $14.25 ....]

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