Warm Up:

Convert $y = -2x^2 + 8x - 10$ to vertex form

$= -2(x^2 - 4x) - 10$

$= -2(x^2 - 4x + (-2)^2 - (-2)^2) - 10$

$= -2(x^2 - 4x + (-2)^2) - (-2)(-2)^2 - 10$

$= -2(x-2)^2 - (-8) - 10$

$= -2(x-2)^2 - 2$

Yes, I know lots of 2s. Sorry!

Solving Word Problems

Lesson objectives

- I know how to identify if the problems are about the zeros, vertex, or y-intercept
- I know how to choose the appropriate tool to find the missing information in the question
- I know how to use the information in previous sections of a question to solve the next part

1.1 Lesson objectives | Teachers' notes | Lesson notes

Nelson Page 178 #s 6 - 14

Problem Solving with Quadratics

When we have a word problem that is centred around a quadratic, we are essentially asking one of three questions:

1. What is the vertex?

   Maximum, minimum problems

2. What are the zeros?

   When will it land?
   How long in the air for?

3. What is the y-intercept?

   Starting height?

The path of a firework is modelled using the equation $h = -5d^2 + 20d + 1$, where h is the height in metres above the ground and d is the horizontal distance, in metres. What is the maximum height of the firework?

a) From what height is the rocket shot into the air?

This is when $d = 0$ (which is the y-intercept)

$\Rightarrow$ 1 metre

b) What is the maximum height of the firework?

$-\frac{b}{2a} = \frac{-20}{2(-5)}$

$= \frac{-20}{-10}$

$= 2$

Find height when $d = 2$

$h = -5(2)^2 + 20(2) + 1$

$= -20 + 40 + 1$

$= 21$ metres

c) How far (horizontally) does the firework travel?

$a = -5, b = 20, c = 1$

$d = \frac{-20 \pm \sqrt{(20)^2 - 4(-5)(1)}}{2(-5)}$

$d = \frac{-20 \pm \sqrt{420}}{-10}$

$d = \frac{-20 + \sqrt{420}}{-10}$

$= -0.049$

$d = \frac{-20 - \sqrt{420}}{-10}$

$= 4.049$

Starts at $d = 0$

Ends at $d = 4.049$

$\Rightarrow$ Travels 4.049m horizontally

An equipment storage shed has a parabolic cross section modelled by the relation $h = -d^2 + 4d$, where h is the height in metres and d is the horizontal distance, in metres, from one edge of the shed.

a) How wide and how tall is the shed.

$h = -d(d-4)$

$-d = 0 \quad d-4=0$

$d = 0 \quad d = 4$

$\Rightarrow$ width $= 4$m

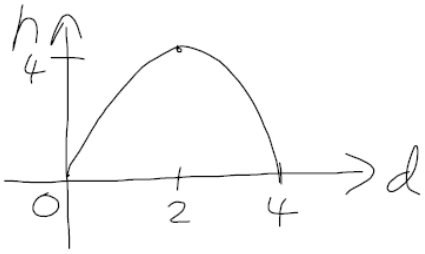
Vertex will be $\frac{0+4}{2}$

$= 2$ m across

$h = -(2)^2 + 4(2)$

$h = -4 + 8 = 4$m tall

b) Sketch the graph.



c) For what values of d is the relation valid? Explain.

Any values between (and including) 0 and 4.
Any other value would give a negative height!

The profit function for a business is given by the equation $P = -4x^2 + 16x - 7$, where x is the number of items sold, in thousands, and P is the profit in thousands of dollars.

a) Calculate the maximum profit and how many items must be sold to achieve it.

$\frac{-b}{2a} = \frac{-16}{2(-4)}$

$= \frac{-16}{-8}$

$= 2$

Sub in $x = 2$

$P = -4(2)^2 + 16(2) - 7$

$P = -16 + 32 - 7$

$P = 9$

$\Rightarrow$ Max profit of \$9000 when selling 2000 items

b) How many items must be sold to break even?

Break even when profit $= 0$

$0 = -4x^2 + 16x - 7$

$ac = (-4)(-7) = 28$

$2 \times 14 = 28$

$2 + 14 = 16$

$-4x^2 + 2x + 14x - 7$

$= -2x(2x-1) + 7(2x-1)$

$= (2x-1)(-2x+7)$

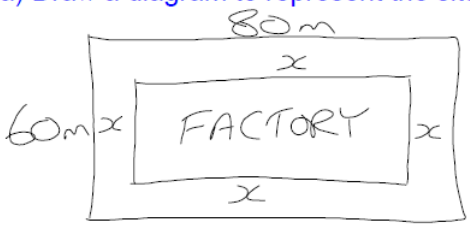
$2x - 1 = 0 \quad -2x + 7 = 0$

$x = \frac{1}{2} \quad x = 3\frac{1}{2}$

Break even when selling 500 (or 3500) items

A factory is to be built on a lot that measures 80m by 60m. A lawn of uniform width, equal to the area of the factory, must surround it.

a) Draw a diagram to represent the situation.



Let the lawn width = x

length of factory = $80-2x$

width of factory = $60-2x$

b) How wide is the strip of lawn, and what are the dimensions of the factory?

Area of lot = 80×60
$= 4800m^2$

Area lawn = Area factory

$\Rightarrow$ area factory = $\frac{1}{2}$ area of lot
$= 2400m^2$

$\Rightarrow$ width of lawn = 10m

length factory = $80-2(10)$
$= 60m$

width factory = $60-2(10)$
$= 40m$

$\Rightarrow (80-2x)(60-2x) = 2400$

$4800-160x-120x+4x^2 = 2400$

$4x^2-280x+2400 = 0$

$x^2-70x+600 = 0$

$(x-10)(x-60) = 0$

$x = 10$ or 60

extraneous (won't work for length or width)