Introduction to Probability Review

Learning Goals

Section	After this section, I can
1.1	• use probability to describe the likelihood of something occurring • measure and calculate simple probabilities
1.2	• calculate theoretical probability
1.3	• recognize the difference between experimental probability and theoretical probability
1.4	• describe how an event can represent a set of probability outcomes • recognize how different events are related • calculate the probability of an event occurring
1.5	• describe and determine how the probability of one event occurring can affect the probability of another event occurring • solve probability problems involving multiple events

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Solutions

1. A mystery spinner is spun several times, producing the results shown in the table.

Colour	Count
Blue	24
Green	48
Yellow	51
Purple	26

a) Calculate the experimental probability of the spinner landing on each colour.

b) Sketch what this spinner could look like. Explain your reasoning.

c) Could the spinner look differently? Explain.

Total trials $= 24+48+51+26$
$= 149$

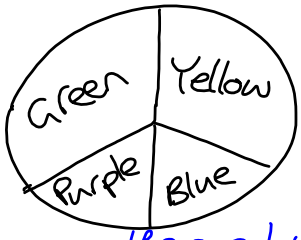
a) $P(\text{Blue}) = \frac{24}{149}$ (16.1%)

$P(\text{Green}) = \frac{48}{149}$ (32.2%)

$P(\text{Yellow}) = \frac{51}{149}$ (34.2%)

$P(\text{Purple}) = \frac{26}{149}$ (17.4%)

b) Probabilities imply that
Blue $\approx 1/6$
Green $\approx 1/3$
Yellow $\approx 1/3$
Purple $\approx 1/6$



c) Yes. These are experimental probabilities, so this is our best guess based on the given info. Further trials may change our opinion: eg there could be another colour on the spinner.

2. A quarterback successfully completed 21 of 35 pass attempts.

a) What is the experimental probability that the quarterback will complete a pass?

b) Suppose the quarterback throws 280 pass attempts over the course of a season. How many is he likely to complete, based on your answer to part a)?

a) $P(\text{completed pass}) = \frac{21}{35}$
$= \frac{3}{5}$

b) # of completed passes $=$ # of attempts $\times P(\text{completed passes})$
$= 280 \times \frac{3}{5}$
$= 168$

3. Match each scenario with its most likely subjective probability. Justify your answers.

	Scenario	Subjective Probability, $P(A)$
a)	Canada will win at least one medal at the next Olympics.	0.1 0.25 0.9
b)	A person selected at random will be left-handed.	
c)	A randomly chosen high school student will be in grade 10.	

a) 0.9 – Canada have an excellent record at the Winter Olympics.

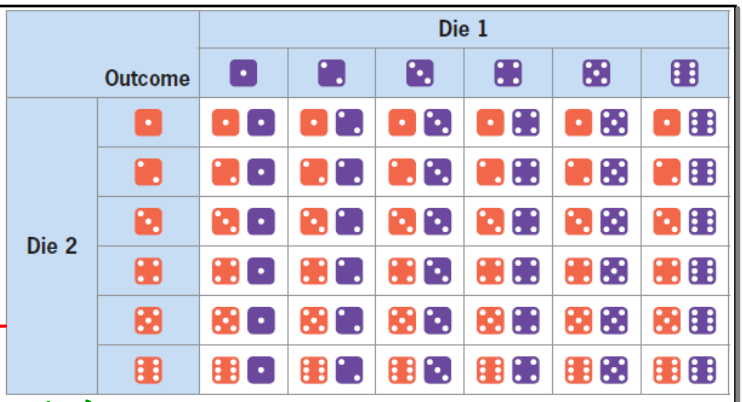
b) 0.1 – About 10% of the world is left-handed.

c) 0.25 – There are 4 grades in high school to choose from.

4. What is the theoretical probability of rolling each of the following sums with a pair of dice?

a) 2 b) 9

c) not 7 d) not a perfect square



a) $P(2) = \frac{1}{36}$

b) $P(9) = \frac{4}{36} = \frac{1}{9}$

c) $P(\text{not } 7) = 1 - P(7)$

$= \frac{36}{36} - \frac{6}{36}$

$= \frac{30}{36}$

$= \frac{5}{6}$

d) $P(\text{not a perfect square})$

$= 1 - P(\text{perfect square})$

$= \frac{36}{36} - \frac{7}{36}$

$= \frac{29}{36}$

Note

4 : (1,3)(2,2)(3,1)

9 : (3,6)(4,5)(5,4)(6,3)

$\Rightarrow$ 7 outcomes

5. A card is randomly drawn from a standard deck of cards. What is the theoretical probability that it will be

a) a club? **b)** an ace?

c) a face card?

a) $P(\text{Club}) = \frac{13}{52}$

$= \frac{1}{4}$

b) $P(\text{Ace}) = \frac{4}{52}$

$= \frac{1}{13}$

c) $P(\text{Face}) = \frac{12}{52}$

$= \frac{3}{13}$

6. A sports analyst predicts that a tennis player has a 25% chance of winning a tournament. What are the odds against winning?

25% chance of winning

$\Rightarrow$ 75% chance of losing

Odds against are

75 : 25

(÷ by 25) 3 : 1

7. A standard die is rolled 24 times and turns up a 3 six times.

a) What is the experimental probability of rolling a 3 on a given trial?

b) What is the theoretical probability of rolling a 3?

c) Explain why these answers are different.

a) $P(3) = \frac{6}{24} = \frac{1}{4}$

b) $P(3) = \frac{1}{6}$

c) Not enough trials (rolls) have been completed to get an accurate answer. The more trials that are completed, the closer the experimental probability comes to equalling the theoretical probability.

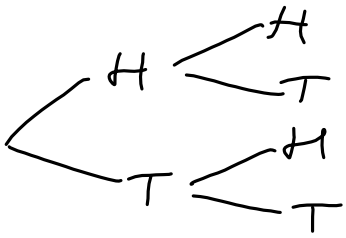
8. Suppose two fair coins are flipped.

a) Draw a tree diagram to illustrate all possible outcomes.

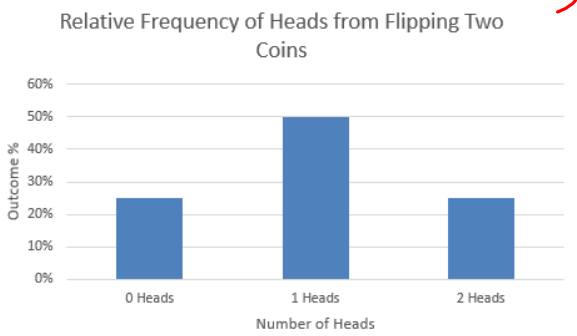
b) Sketch a bar graph that shows the predicted relative frequency of each of the following events when a very large number of trials is carried out
- no heads
- one head
- two heads

c) Explain why your graph has the shape that it does.

a)



b)



c) From the tree diagram

$P(0 \text{ Heads}) = \frac{1}{4}$

$P(1 \text{ Head}) = \frac{1}{2}$

$P(2 \text{ Heads}) = \frac{1}{4}$

We would expect 0 Heads and 2 Heads to be similar and 1 Head to be double the two.

9. A graphing calculator is programmed to randomly generate an integer value between 1 and 8. Determine the probability that the number will be

a) a five or an eight

b) a prime number or a perfect square

c) an even number or a seven

d) not a composite number or an odd number

	1	2	3	4	5	6	7	8
Prime		X	X		X		X	
Square	X			X				
Even		X		X		X		X
Composite				X		X		X
Odd	X		X		X		X	

Note:

Composite numbers are numbers that are not prime or one

Prime numbers only have two factors (themselves and one)

Square numbers are numbers are the result of multiplying integers (whole numbers) by themselves

a) $P(5 \text{ or } 8) = P(5) + P(8)$

$= \frac{1}{8} + \frac{1}{8}$

$= \frac{2}{8} = \frac{1}{4}$

b) $P(\text{Prime or Square}) = P(\text{Prime}) + P(\text{Square})$

$= \frac{4}{8} + \frac{2}{8}$

$= \frac{6}{8} = \frac{3}{4}$

c) $P(\text{Even or 7}) = P(\text{Even}) + P(7)$

$= \frac{4}{8} + \frac{1}{8}$

$= \frac{5}{8}$

d) $P(\text{Not comp or Odd}) = P(\text{not comp}) + P(\text{Odd}) - P(\text{not comp and odd})$

$= \frac{5}{8} + \frac{4}{8} - \frac{4}{8}$

$= \frac{5}{8}$

10. A small vehicle rental company randomly assigns its vehicles to customers based on whatever happens to be available. The fleet is shown below.

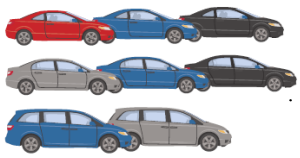
Assume that each vehicle has an equal probability of being available at any given time. Determine the probability that a customer will randomly be assigned:

a) a coupe or a mini-van

b) a blue vehicle or a mini-van

c) a grey vehicle or a sedan

d) not a red vehicle or a coupe



A *coupe* is a car with two passenger doors.
A *mini-van* is larger than a car but smaller than a van.
A *sedan* is a car with four passenger doors.

a) $P(\text{C or MV}) = P(\text{C}) + P(\text{MV})$

$= \frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

b) $P(\text{Blue or MV}) = P(\text{Blue}) + P(\text{MV}) - P(\text{B and MV})$

$= \frac{3}{8} + \frac{2}{8} - \frac{1}{8}$

$= \frac{4}{8} = \frac{1}{2}$

c) $P(\text{Grey or S}) = P(\text{grey}) + P(\text{S}) - P(\text{Grey and S})$

$= \frac{2}{8} + \frac{3}{8} - \frac{1}{8}$

$= \frac{4}{8} = \frac{1}{2}$

d) $P(\text{C or not Red}) = P(\text{C}) + P(\text{not Red}) - P(\text{C and not Red})$

$= \frac{3}{8} + \frac{7}{8} - \frac{2}{8}$

$= \frac{8}{8} = 1$ (certain!)

11. A standard die is rolled and a card is drawn from a standard deck of playing cards.

a) Which of the following is more likely to occur?
- an even value will be rolled and a heart will be drawn
- a composite value will be rolled and a face card will be drawn

b) Justify your answer with mathematical reasoning.

a) $P(\text{Even and Heart}) = P(\text{Even}) \times P(\text{Heart})$

$= \frac{3}{6} \times \frac{13}{52}$

$= \frac{39}{312} = \frac{1}{8}$

$P(\text{Composite and face}) = P(\text{composite}) \times P(\text{face})$

$= \frac{2}{6} \times \frac{12}{52}$

$= \frac{24}{312} = \frac{1}{13}$

b) Even and heart is more likely because $\frac{1}{8}$ (12.5%) is greater than $\frac{1}{13}$ (7.7%)

12. A bag has 3 red tiles, 1 yellow tile, and 2 green tiles.

a) What is the probability that a red tile is drawn, followed by a second red tile, if the first tile is replaced?

b) How does this value change if the first tile drawn is not replaced?

c) Explain why these answers are different.

Total # of tiles $= 3+1+2$

$= 6$ tiles

a) $P(\text{Red, Red}) = P(\text{Red}) \times P(\text{Red})$

$= \frac{3}{6} \times \frac{3}{6}$

$= \frac{9}{36} = \frac{1}{4}$

b) $P(RR) = P(R) \times P(R|R)$

$= \frac{3}{6} \times \frac{2}{5}$

$= \frac{6}{30} = \frac{1}{5}$

c) If you don't replace the tile the sample space changes (6 to 5) and the number of red outcomes changes as well (3 to 2).

13. Josiah has a 20% experimental probability of hitting the snooze button any morning when his alarm goes off. When he hits the snooze button, there is a 25% conditional probability that he misses his bus. He has never missed the bus when he has not hit the snooze button. If Josiah's alarm woke him 120 times over the course of the semester, how many times did Josiah miss his bus?

$P(\text{miss bus}) = P(\text{snooze}) \times P(\text{miss} \mid \text{snooze})$

$= 0.20 \times 0.25$

$= 0.05$

$\Rightarrow$ # of missed buses $=$ # of alarms $\times P(\text{miss bus})$

$= 120(0.05)$

$= 6$ times