Warm Up:

Given $f(x)=2(x-1)^2-5$, state the number of solutions and then determine the solutions.

a and k are opposite signs ⟹ 2 solutions

$0 = 2(x-1)^2 - 5$

$\frac{5}{2} = \frac{2(x-1)^2}{2}$

$\frac{5}{2} = (x-1)^2$

$\pm\sqrt{\frac{5}{2}} = x-1$

$1 \pm \sqrt{\frac{5}{2}} = x$

$1+\sqrt{\frac{5}{2}} = 2.581$

$1-\sqrt{\frac{5}{2}} = -0.581$

Max and Min Values of a Quadratic

Lesson objectives

- I know how to identify from any form of the equation if the quadratic will have a maximum or a minimum value
- I know how to identify the maximum or minimum value from any form of the equation

1.1

Lesson objectives | Teachers' notes | Lesson notes

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Determining Maximum and Minimum Values

- We have seen before that the value of "a" tells us if the parabola opens up or down
- A parabola that opens up has a minimum value
- A parabola that opens down has a maximum value
- When we are asked to find a maximum value we are looking for the y-value of the vertex

Finding the Vertex

1. Vertex Form $y = a(x - h)^2 + k$

$$V = (h, k)$$

** Remember - change the sign of h!

2. Standard Form $y = ax^2 + bx + c$

We have a short cut to find the x value of the vertex from standard form - we can use

$$h = \frac{-b}{2a}$$

We can then use that value of x to find the corresponding value of y. (k-value)

3. Factored Form $y = a(x - r)(x - s)$

To find the x value of the vertex we need to first find the zeros.

We then add the zeros together and divide by 2 to find the middle, which is the x-value of the vertex.

To find the y value, use x in the equation.

$$x = \frac{(r+s)}{2}$$

Example

State whether the following functions have max ormin values and then find the values.

a) $f(x) = -x^2 + 7x$

a is negative

$\Rightarrow$ maximum

$h = \frac{-b}{2a} = \frac{-7}{2(-1)} = \frac{-7}{-2} = 3\frac{1}{2}$

$k = -(3\frac{1}{2})^2 + 7x$

$k = 12.25$

b) $f(x) = -4(x+2)(x-3)$

a is negative $\Rightarrow$ max

$r = -2,\ s = 3$

$h = \frac{-2+3}{2} = \frac{1}{2}$

$k = -4(\frac{1}{2}+2)(\frac{1}{2}-3)$

$k = 25$

c) $g(x) = 3(x-1)^2 - 4$

a is positive

$\Rightarrow$ minimum

$k = -4$

d) $h(x) = 4x^2 + 3x - 5$

a is positive $\Rightarrow$ minimum

$h = \frac{-b}{2a} = \frac{-3}{2(4)} = \frac{-3}{8}$

$k = 4(\frac{-3}{8})^2 + 3(\frac{-3}{8}) - 5$

$k = -5.5625$ $(-5\frac{9}{16})$

Converting to Vertex Form

To convert to vertex form we can complete the square, but that is a lot of complicated work!
So the short cut is to find the vertex and use the value of "a" from the original question!
We then plug all the information into vertex form.

a) $f(t) = -5t^2 + 40t + 100$

$h = \frac{-b}{2a} = \frac{-40}{2(-5)} = 4$

$k = -5(4)^2 + 40(4) + 100$

$k = 180$

$\Rightarrow -5(t-4)^2 + 180$

b) $g(x) = 3x^2 + 9x - 2$

$h = \frac{-b}{2a} = \frac{-9}{2(3)} = \frac{-9}{6} = \frac{-3}{2}$

$k = 3\left(-\frac{3}{2}\right)^2 + 9\left(-\frac{3}{2}\right) - 2$

$k = -8\frac{3}{4}$

$\Rightarrow 3\left(x+\frac{3}{2}\right)^2 - 8\frac{3}{4}$

c) $h(n) = 2n^2 + 5n + 3$

$h = \frac{-b}{2a} = \frac{-5}{2(2)} = \frac{-5}{4}$

$k = 2\left(-\frac{5}{4}\right)^2 + 5\left(-\frac{5}{4}\right) + 3$

$k = -\frac{1}{8}$

$\Rightarrow 2\left(x+\frac{5}{4}\right)^2 - \frac{1}{8}$

d) $k(x) = 3(x-2)(x-4)$

$h = \frac{r+s}{2} = \frac{2+4}{2} = \frac{6}{2} = 3$

$k = 3(3-2)(3-4)$

$k = -3$

$\Rightarrow 3(x-3)^2 - 3$

Example

The height of a ball thrown vertically upward from a roof top is modelled by h(t) = -5t² + 20t + 50, where h(t) is the ball's height above the ground in metres, at time t seconds after it is thrown.

a) How long does it take for the ball to reach its maximum height?

$h = \frac{-b}{2a} = \frac{-20}{2(-5)} = \frac{-20}{-10} = 2$ seconds

b) Determine the max height of the ball.

$k = -5(2)^2 + 20(2) + 50$

$k = -20 + 40 + 50$

$k = 70$m

c) How high is the roof top?

$c = 50$m