Independent and Dependent Events

Lesson objectives

- I can describe and determine how the probability of one event occurring can affect the probability of another event occurring
- I can solve probability problems involving multiple events

1.1

Lesson objectives | Teachers' notes | Lesson notes

MHR Page 53 #s 1 - 8

Warm Up

Modern technology allows doctors to predict the gender of a baby long before it is born. Some parents like to know this information as soon as possible, while others prefer to keep it a surprise.

Look at the family pictured.

- Do you think having three boys and three girls is very likely?
- Do you think the gender of one birth will have an effect on the gender of a following birth? Why or why not?

When multiple events occur in a probability experiment, these events are called compound events.

Discuss with the person next to you and be prepared to share your answers.

Definitions

Compound Events

- Multiple events in a probability experiment
- May or may not effect each other

Independent Events

- Situations in which the occurrence or non-occurrence of one event has no influence on the the probability of the other event occurring

Dependent Events

- The occurrence or non-occurrence of one event influences the probability of the other event occurring

Conditional Probability

- Probability of a second event occurring, given that a first event occurred
- The sample space for the second event is reduced from the first event

Example 1

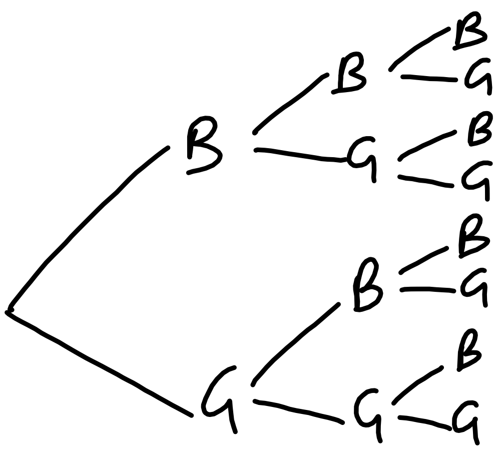
Simple Compound Events

a) The Archers currently have no children, but would like to have three children. Assuming that they do have three children and there is an equal probability of any birth resulting in a boy or a girl, what is the probability that they will all be boys?

b) The Burnell family has two sons. What is the probability that their third child will be a boy?

c) Why are the answers to a) and b) different?

a)



$P(BBB) = \frac{1}{8}$

b) $P(B) = \frac{1}{2}$

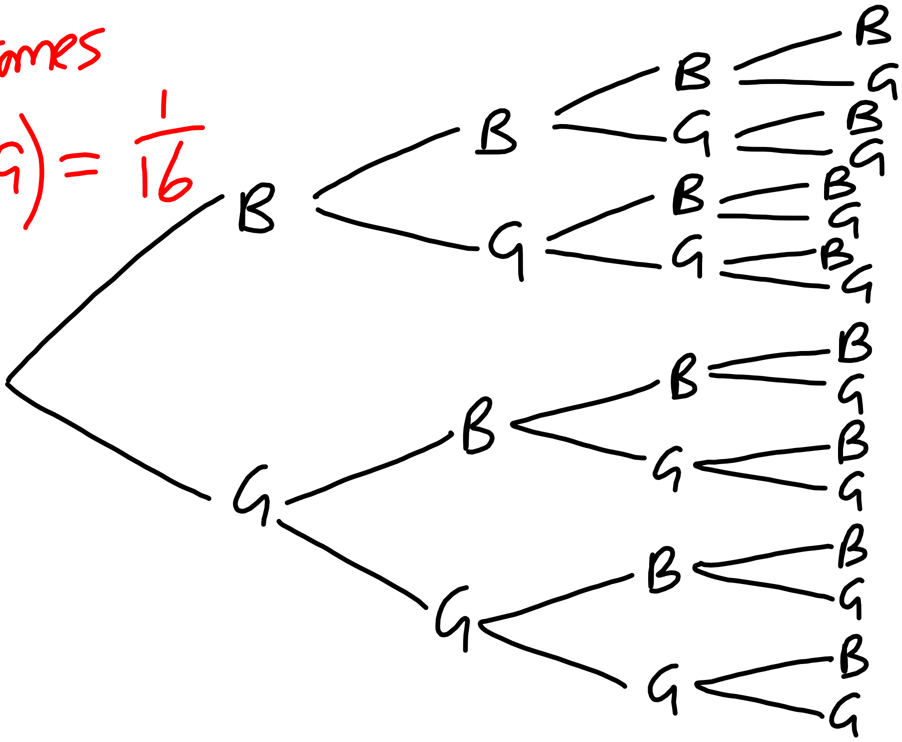
c) (b) is a multiple event, (a) is a single event

Your Turn

The Singh family currently has no children but hopes to have four children. Assuming they do have four children, what is the probability that they will all be girls?

16 outcomes

$P(GGGG) = \frac{1}{16}$



Sometimes the probability of one event depends on whether another event occurs, and sometimes it does not. When one event has no effect on the probability of another, the events are **independent**.

Example 2

Probability of Independent Events

Olivia has four highlighting pens in her pencil case: two yellow, one orange, and one blue. She reaches into her pencil case and randomly chooses a highlighter. After she uses it, she immediately replaces it in the case so it can be used again. What is the probability that she will choose

a) two yellow highlighters in a row?

b) a yellow highlighter followed by a blue highlighter?

$\Rightarrow$ Replacement of pens, so the events are INDEPENDENT

a) $P(YY) = P(Y) \times P(Y)$

$= \frac{2}{4} \times \frac{2}{4}$

$= \frac{4}{16}$

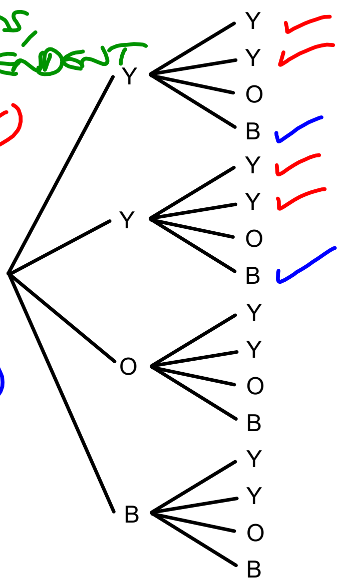
$= \frac{1}{4}$

b) $P(YB) = P(Y) \times P(B)$

$= \frac{2}{4} \times \frac{1}{4}$

$= \frac{2}{16}$

$= \frac{1}{8}$



Your Turn

Three green marbles and two yellow marbles are placed into a bag. What is the probability of randomly drawing a green marble followed by a yellow marble, assuming that the first marble is replaced before the second marble is drawn?

Again we have replacement, so they are INDEPENDENT.

$$P(GY) = P(G) \times P(Y)$$
$$= \frac{3}{5} \times \frac{2}{5}$$
$$= \frac{6}{25}$$

		2nd Pick				
		G	G	G	Y	Y
1st Pick	G	GG	GG	GG	GY	GY
	G	GG	GG	GG	GY	GY
	G	GG	GG	GG	GY	GY
	Y	YG	YG	YG	YY	YY
	Y	YG	YG	YG	YY	YY

Another way to determine the probability of compound independent events is to multiply the probability of the first event by the probability of the second event. In Example 2, the probability of Olivia choosing a yellow highlighter is 2 out of 4 or $\frac{1}{2}$, and the probability of choosing a blue highlighter is 1 out of 4 or $\frac{1}{4}$.

Using this method, the probability of choosing two yellow highlighters in a row is

$$P(YY) = \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{4}$$

Similarly, calculate the probability of choosing a yellow highlighter followed by a blue highlighter.

$$P(YB) = \frac{1}{2} \times \frac{1}{4}$$
$$= \frac{1}{8}$$

Note that these values agree with the ones obtained from analysing the tree diagram.

This result is an illustration of the multiplicative principle for independent events.

Multiplicative Principle (Fundamental Counting Principle) for Independent Events

The probability of two independent events, A and B, occurring is

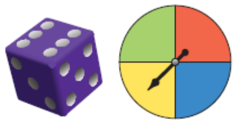
$P(A \text{ and } B) = P(A) \times P(B)$

Example 3

Different Compound Events

A game is played in which a standard die is rolled and a spinner is spun.

Player A wins a point if the spinner lands on red and an even number is rolled. Player B wins a point if the spinner lands on yellow or green and a composite number is rolled. Is this a fair game? Explain.



$$P(R \text{ and Even}) = P(R) \times P(\text{Even})$$
$$= \frac{1}{4} \times \frac{3}{6}$$
$$= \frac{3}{24} = \frac{1}{8}$$

$$P(Y \text{ or } G \text{ and Composite})$$
$$= P(Y \text{ or } G) \times P(\text{Composite})$$
$$= \frac{2}{4} \times \frac{2}{6}$$
$$= \frac{4}{24} = \frac{1}{6}$$

NOTE

Composite #s are non-prime positive integers and not 1.

$\Rightarrow$ Game is not fair. Player B has a greater chance of winning

Your Turn

What is the probability of flipping heads with a fair coin and rolling a prime number with a fair die?

$$P(H \text{ and Prime}) = P(H) \times P(\text{Prime})$$
$$= \frac{1}{2} \times \frac{3}{6}$$
$$= \frac{3}{12}$$
$$= \frac{1}{4}$$

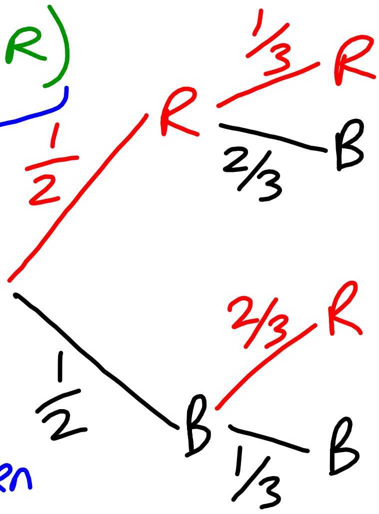
Sometimes the probability of one event occurring has an effect on another event occurring. When this happens, the events are said to be dependent.

Example 4

Probability of Dependent Events

Two red checkers and two black checkers are placed into a bag. What is the probability that a red checker is randomly chosen, followed by a second red checker, assuming that the first checker drawn is not replaced?

$$P(RR) = P(R) \times \underbrace{P(R|R)}$$
$$= \frac{2}{4} \times \frac{1}{3}$$
$$= \frac{2}{12}$$
$$= \frac{1}{6}$$



Probability of a red given that a red has already been chosen.

Your Turn

A bag contains two apples, one orange, and two peaches. Suppose Jelena reaches in and chooses a piece of fruit at random, and then selects another piece of fruit without replacing the first one. What is the probability that she will choose two peaches?

$$P(PP) = P(P) \times P(P|P)$$
$$= \frac{2}{5} \times \frac{1}{4}$$
$$= \frac{2}{20}$$
$$= \frac{1}{10}$$

The previous example illustrates how the outcome of one trial can affect the probable outcome of a subsequent trial. When the first checker was chosen and not replaced the sample space was reduced, which changed the probability values for the second trial.

When the outcome of one trial has been determined, then the **conditional probability** of a subsequent trial can be calculated based on the result of the first trial. If the first event is A and the second event is B, then $P(B|A)$ represents the conditional probability that B will occur, given that A has occurred. To calculate the probability of both dependent events occurring, apply the multiplicative principle for dependent events.

Multiplicative Principle for Dependent Events

To calculate the probability of two dependent events, A and B, occurring, multiply the probability that A occurs by the conditional probability that B occurs, given that A occurred.

$P(A \text{ and } B) = P(A) \times P(B|A)$

Example 5

Conditional Probability in Telemarketing

A study of a telemarketing company's data showed that of 1000 calls placed:

- The experimental probability of a call receiver staying on the line for at least a minute was 16%.
- The conditional probability of a call resulting in a sale, given that a receiver stayed on the line for at least a minute, was 10%.
- No sales were made if the receiver did not stay on the line for at least a minute.

How many sales were made?

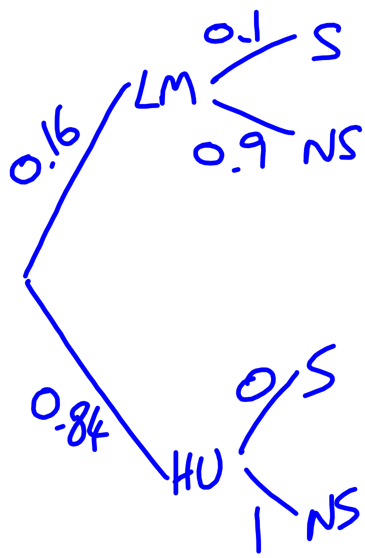
$$P(\text{sale}) = P(\text{at least a minute}) \times P(\text{sale} \mid \text{least a min})$$

$$= 0.16 \times 0.1$$

$$= 0.016$$

$$\Rightarrow 1000 \text{ calls} = 1000(0.016)$$

$$= 16 \text{ sales}$$



Your Turn

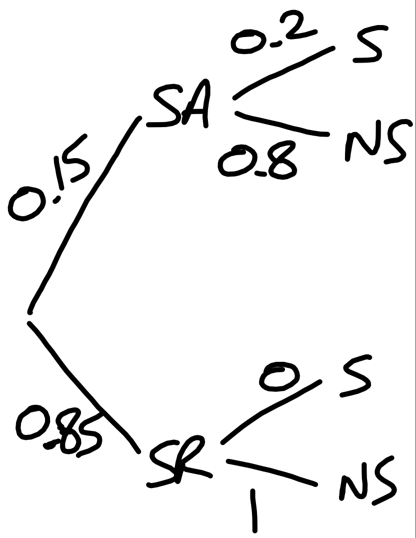
Lars is offering juice samples at a shopping mall. The experimental probability of a randomly chosen shopper accepting a sample is 15%. The conditional probability of a customer purchasing some juice given that he or she tried a sample is 20%. No one purchases juice without trying a sample. If Lars offers 500 people juice samples, how many sales will he make?

$$P(\text{sale}) = P(\text{accept sample}) \times P(\text{sale} \mid \text{accept})$$

$$= 0.15 \times 0.20$$

$$= 0.03$$

$$\Rightarrow 500 \text{ offers} = 500(0.03) = 15 \text{ sales}$$



R1. a) What is the difference between independent events and dependent events?

b) Provide an example of each.

a) Independent events have no influence on each other's probability of occurring, while dependent events do influence the probability of the other event occurring.

b) Drawing two cards from a deck with replacement versus without replacement.

R2. Which of the following scenarios is most likely to occur, and why?

- a coin is flipped three times and comes up heads every time
- after coming up heads four times, a coin comes up heads on the fifth toss

The second scenario is more likely because the first scenario involves multiple events while the second only involves a single event.

$P(HHH) = (0.5)^3 = 0.125$ versus $P(H \text{ on 5th flip}) = 0.5$

R3. **a)** Explain what is meant by conditional probability.

b) Describe a situation in which conditional probability
- applies
- does not apply

a) Conditional probability is the probability of a second event occurring, given that a first event occurred.

b) There are four red marbles and three blue counters in a bag. Conditional - Probability of choosing a red given that you already choose a blue. Not conditional - Probability of choosing a blue on your second pick if the first choice is replaced.

R4. What are some advantages of using a probability tree diagram in solving problems involving dependent events?

Helps to identify all the favourable outcomes as well as the total number of outcomes.