Warm Up:

Find the roots of the following equation.

Hint: Factor!

$y = x^2 + 5x + 6$

$= (x+2)(x+3)$

$x+2=0 \quad x+3=0$

$x=-2 \quad x=-3$

$y = 2x^2 + 9x + 4$

$ac = (2)(4) = 8$

$1 \times 8 = 8$

$1+8=9$

$= 2x^2 + x + 8x + 4$

$= x(2x+1) + 4(2x+1)$

$= (2x+1)(x+4)$

$2x+1=0 \quad x+4=0$

$x = -\frac{1}{2} \quad x = -4$

Zeros of a Quadratic

Lesson objectives

- I know how to find the zeros of a quadratic from factored form
- I know how to use the quadratic formula to find the zeros of a quadratic that can't be factored
- I know how to determine the number of zeros of a quadratic without finding the zeros

1.1 Lesson objectives | Teachers' notes | Lesson notes

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Finding the Solution

To find the solution to a quadratic equation weneed to rearrange it so that we have it in standard form. Then we factor. When we set each factor equal to zero and solve, this gives us our solutions.

$2x^2 + 3x + 2 = 1$

Make equation = 0

$2x^2 + 3x + 1 = 0$

$ac = (2)(1) = 2$

$1 \times 2 = 2$

$1 + 2 = 3$

$\Rightarrow 2x^2 + x + 2x + 1 = 0$

$x(2x+1) + 1(2x+1) = 0$

$(2x+1)(x+1) = 0$

$2x + 1 = 0$ $\quad x + 1 = 0$

$x = -\frac{1}{2}$ $\quad x = -1$

$-5x^2 + 5x + 40 = 10$

[Get all terms on one side]

$0 = 5x^2 - 5x - 30$

Common factor of 5

$\Rightarrow 0 = x^2 - x - 6$

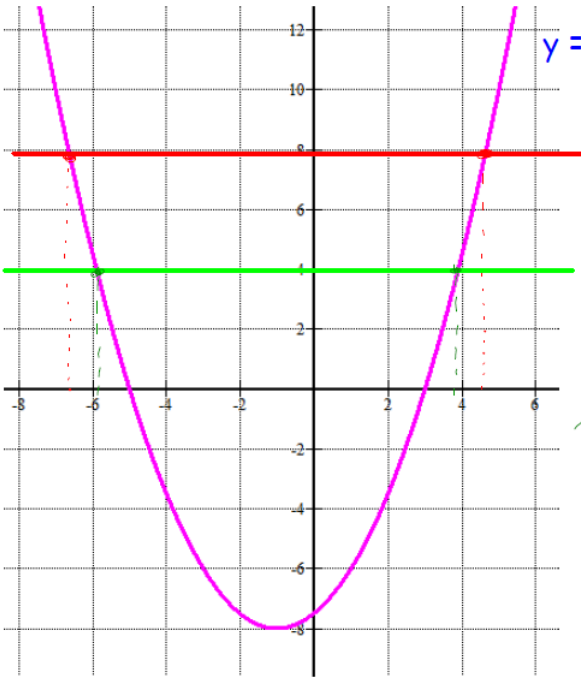
$0 = (x-3)(x+2)$

$x - 3 = 0$ $\quad x + 2 = 0$

$x = 3$ $\quad x = -2$

Graphical Interpretations of Solutions

The graphical interpretation of the solution to a quadratic occurs if you draw a horizontal line across the parabola at the desired value of y.



If $y = 8$

then $x \simeq -6.5$

or $x \simeq 4.5$

If $y = 4$

then $x \simeq -5.9$

or $x \simeq 3.9$

Solving Equations

If we can't find numbers to factor an equation doesthat mean there are no solutions?

Does the equatior $y = x^2 + 6x + 1$ have any x-intercepts?

Where is the vertex?

$y = (x^2 + 6x + 3^2 - 3^2) + 1$

$y = (x^2 + 6x + 3^2) - 3^2 + 1$

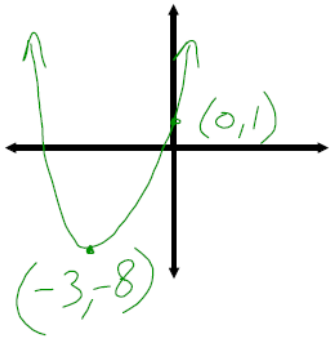
$y = (x + 3)^2 - 8$

So the vertex is below the y-axis at (-3,-8).

Does it open up or down?

The parabola opens up.

Sketch the parabola.



Solving Equations

Therefore, the parabola has to cross the x-axisso there must be solutions.

How do we find them?

The Quadratic Formula!

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

What are the solutions to $0 = x^2 + 6x + 1$?

$a = 1, b = 6, c = 1$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{36 - 4}}{2}$$

press = before dividing

$$x = \frac{-6 + \sqrt{32}}{2}$$

$$x = \frac{-6 - \sqrt{32}}{2}$$

$x = -0.172$ or -5.828

1. Use the quadratic formula to solve each equation. Express answers as exact roots.

b) $2x^2 + 4x - 7 = 0$

c) $4x^2 - 12x + 9 = 0$

2. **Use Technology** Use the quadratic formula to solve. Express your answers as exact roots and as approximate roots, rounded to the nearest hundredth. Verify graphically with technology.

d) $10x^2 - 45x - 7 = 0$

e) $-5x^2 + 16x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. Use the quadratic formula to solve each equation. Express answers as exact roots.

b) $2x^2 + 4x - 7 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 2, b = 4, c = -7$

$$x = \frac{-4 \pm \sqrt{(4^2 - 4(2)(-7))}}{2(2)}$$

$$x = \frac{-4 \pm \sqrt{(16 - -56)}}{4}$$

$$\mathbf{x = \frac{-4 \pm \sqrt{(72)}}{4}}$$

We can't find an exact square root of 72, so this is the exact answer.

1. Use the quadratic formula to solve each equation. Express answers as exact roots.

c) $4x^2 - 12x + 9 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a = 4, b = -12, c = 9

$$x = \frac{12 \pm \sqrt{((-12)^2 - 4(4)(9))}}{2(4)}$$

$$x = \frac{12 \pm \sqrt{(144 - 144)}}{8}$$

$$x = \frac{12 \pm \sqrt{(0)}}{8}$$

$$x = \frac{12 + \sqrt{(0)}}{8} \qquad x = \frac{12 - \sqrt{(0)}}{8}$$

$$x = \frac{3}{2} \qquad x = \frac{3}{2}$$

This gives a repeated solution. It means that the solution is also the vertex.

2. **Use Technology** Use the quadratic formula to solve. Express your answers as exact roots and as approximate roots, rounded to the nearest hundredth. Verify graphically with technology.

d) $10x^2 - 45x - 7 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a = 10, b = -45, c = -7

$$x = \frac{45 \pm \sqrt{((-45)^2 - 4(10)(-7))}}{2(10)}$$

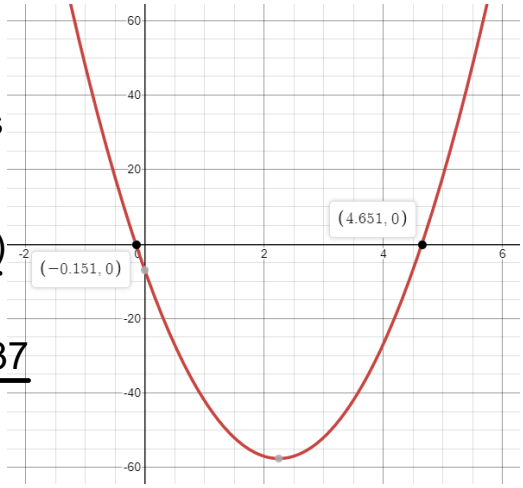
$$x = \frac{45 \pm \sqrt{(2025 - -280)}}{20}$$

$$x = \frac{45 \pm \sqrt{(2305)}}{20}$$ ← Exact answers

$$x = \frac{45 + \sqrt{(2305)}}{20} \qquad x = \frac{45 - \sqrt{(2305)}}{20}$$

$$x = \frac{93.01041554}{20} \qquad x = \frac{-3.010415537}{20}$$

$x = 4.65$ $\qquad$ $x = -0.15$ ← Approximate answers



2. **Use Technology** Use the quadratic formula to solve. Express your answers as exact roots and as approximate roots, rounded to the nearest hundredth. Verify graphically with technology.

e) $-5x^2 + 16x - 2 = 0$

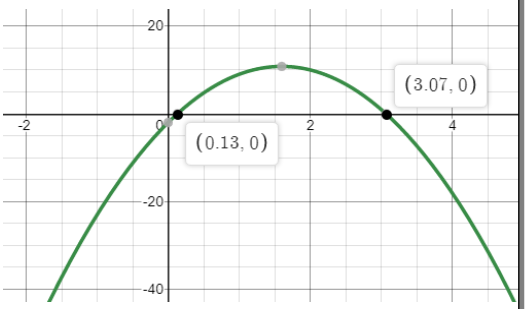
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a = -5, b = 16, c = -2

$$x = \frac{-16 \pm \sqrt{(16^2 - 4(-5)(-2))}}{2(-5)}$$

$$x = \frac{-16 \pm \sqrt{(256 - 40)}}{-10}$$

$$\mathbf{x = \frac{-16 \pm \sqrt{(216)}}{-10}}$$ ← Exact answers



$$x = \frac{-16 + \sqrt{(216)}}{-10}$$

$$x = \frac{-1.303061543}{-10}$$

$$\mathbf{x = 0.13}$$

$$x = \frac{-16 - \sqrt{(216)}}{-10}$$

$$x = \frac{-30.69693846}{-10}$$

$$\mathbf{x = 3.07}$$

← Approximate answers

Factoring vs Quadratic Formula

Factoring is the most efficient way to find the solution to a quadratic equation.

The Quadratic Formula will work every time, but is not the most efficient method. It can also be easy to make algebraic errors, so factoring is the better option.

Number of Solutions

How many solutions can we have to a quadratic?

This is the same question as how many x-intercepts can a quadratic have?

The DISCRIMINANT - the expression under the square root sign in the quadratic formula ($b^2 - 4ac$)

2 solutions:
the discriminant is greater than 0.

1 solution:
the discriminant is equal to 0.

0 solutions:
the discriminant is less than 0.

Solving from Vertex Form

When we have vertex form and need to find the solutions, it is the **ONLY** time we can isolate a quadratic!

We need to make sure we are doing inverse operations in the proper order.

When we take a square root we need to take the positive and the negative square root.

Example

Find the solution when y = 4

$y = 4(x-3)^2 + 2$

$4 = 4(x-3)^2 + 2$

$4-2 = 4(x-3)^2$

$\frac{2}{4} = \frac{4(x-3)^2}{4}$

$\frac{1}{2} = (x-3)^2$

$\pm\sqrt{\frac{1}{2}} = \sqrt{(x-3)^2}$

$\pm\sqrt{\frac{1}{2}} = x-3$

$3 \pm \sqrt{\frac{1}{2}} = x$

$x = 3+\sqrt{\frac{1}{2}} = 3.707$

$x = 3-\sqrt{\frac{1}{2}} = 2.293$

$y = -(x+1)^2 - 3$

$4 = -(x+1)^2 - 3$

$4+3 = -(x+1)^2$

$\frac{7}{-1} = \frac{-(x+1)^2}{-1}$

$-7 = (x+1)^2$

$\sqrt{-7}$ = no solution! can't find the square root of a negative #.

2. Use two different methods to find the roots of $2(x-3)^2 - 11 = 0$ to two decimal places. Hint: convert to standard form and then use the quadratic formula is one way.

3. Find the roots of the following, if possible. Use the most appropriate method.

a) $x^2 - 8x = -16$

b) $2x^2 + 3x - 20 = 0$

c) $(x-5)^2 = 16$

d) $x^2 + 10 = 0$

e) $-2(x+1)^2 + 10 = 0$

f) $x^2 = 90 - 6x$

g) $-5x^2 + 15x = 11$

h) $3.2w^2 + 28.9w - 8.4 = 0$

i) $-4.9(t-4)^2 + 50 = 0$

2. Use two different methods to find the roots of $2(x-3)^2-11=0$ to two decimal places. Hint: convert to standard form and then use the quadratic formula is one way.

Using SAMDEB

$2(x-3)^2 - 11 = 0$

$\frac{2(x-3)^2}{2} = \frac{11}{2}$

$(x-3)^2 = 5.5$

$x - 3 = \pm\sqrt{5.5}$

$x = 3 \pm \sqrt{5.5}$

$x = 5.35$ or 0.65

First expand and simplify

$2(x-3)(x-3) - 11 = 0$

$2(x^2 - 3x - 3x + 9) - 11 = 0$

$2(x^2 - 6x + 9) - 11 = 0$

$2x^2 - 12x + 18 - 11 = 0$

$2x^2 - 12x + 7 = 0$

Then use the quadratic formula

$a = 2, b = -12, c = 7$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(2)(7)}}{2(2)}$$

$$x = \frac{12 \pm \sqrt{144 - (56)}}{4}$$

$$x = \frac{12 \pm \sqrt{88}}{4}$$

$x = 5.35$ or 0.65

3. Find the roots of the following, if possible. Use the most appropriate method.

c) $(x-5)^2 = 16$

$(x - 5)^2 = 16$

$x - 5 = \pm\sqrt{16}$

$x = 5 \pm \sqrt{16}$

$x = 9$ or 1

3. Find the roots of the following, if possible. Use the most appropriate method.

e) $-2(x+1)^2+10=0$

$-2(x+1)^2+10=0$

$\frac{-2(x+1)^2}{-2}=\frac{-10}{-2}$

$(x+1)^2=5$

$x+1=\pm\sqrt{5}$

$x=-1\pm\sqrt{5}$

$x=1.236$ or -3.236

3. Find the roots of the following, if possible. Use the most appropriate method.

i) $-4.9(t-4)^2+50=0$

$-4.9(t-4)^2+50=0$

$\frac{-4.9(t-4)^2}{-4.9}=\frac{-50}{-4.9}$

$(t-4)^2=10.20408$

$t-4=\pm\sqrt{10.20408}$

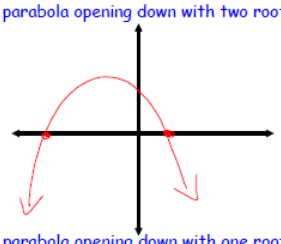
$t=4\pm\sqrt{10.20408}$

$t=7.194$ or 0.806

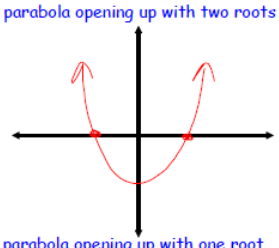
Determining the Number of Zeros from Vertex Form

Quickly sketch the following:

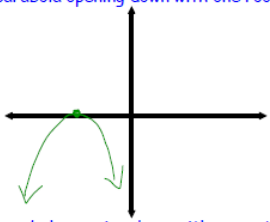
a parabola opening down with two roots



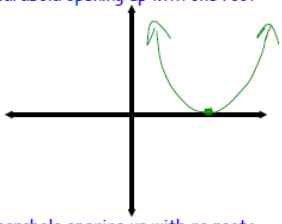
a parabola opening up with two roots



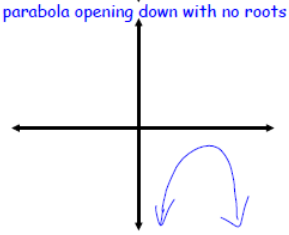
a parabola opening down with one root



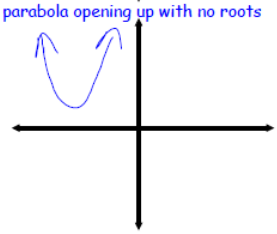
a parabola opening up with one root



a parabola opening down with no roots



a parabola opening up with no roots



Compare the approximate values of a and k in each case. Can you conclude anything that would lead you to a rule about the number of zeros?

Number of Zeros in Vertex Form

If a and k have the same sign there are no zeros

If a and k have opposite signs there are two zeros

If k = 0, there is one zero

Examples

$y = 2(x-4)^2 + 5$

a and k = same sign

$\Rightarrow$ no zeros

$y = -3(x-1)^2 + 5$

a and k = opposite signs

$\Rightarrow$ two zeros

$y = 2(x+1)^2$

$k = 0$

$\Rightarrow$ one zero

$y = -3(x)^2 - 4$

a and k = same signs

$\Rightarrow$ no zeros