Mutually and Non-Mutually Exclusive Events

Lesson objectives

- I can describe how an event can represent a set of probability outcomes
- I can recognise how different events are related
- I can calculate the probability of an event occurring

1.1

Lesson objectives | Teachers' notes | Lesson notes

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Warm Up

Playing cards has been a popular source of entertainment for hundreds of years. Simple games such as Snap and Crazy Eights can be easily learned by young children. More complex strategy-based games such as bridge can be more challenging.

- What are some card games that you have heard of?
- What makes them interesting or fun to play?
- How are the games related to probability?

Discuss with the person next to you and be prepared to share your answers.

Definitions

Mutually Exclusive Events

- Events that have different attributes
- Cannot occur simultaneously

Non-Mutually Exclusive Events

- Different events that can happen at the same time

When calculating the probability of either one event happening or another happening, it is important to carefully count the outcomes. This is relatively easy to do when the events have completely different characteristics. Such events are said to be **mutually exclusive events**, because you can have either one event or the other, but not both.

When you flip a coin, it comes up either heads or tails. It cannot be both. Similarly, a newborn is either a boy or a girl. When a card is drawn from a standard deck of cards, it will be a club, a diamond, a heart, or a spade. All of these are examples of mutually exclusive events.

Example 1

Probability of Mutually Exclusive Events

A picnic basket of sandwiches contains 3 ham, 2 turkey, 1 chicken, and 4 egg salad sandwiches. What is the probability of reaching in and randomly choosing either a ham or an egg salad sandwich?

Total of $3+2+1+4 = 10$ sandwiches

3 ham and 4 egg salad

$\Rightarrow$ 7 favourable outcomes

$$P(\text{"winning"}) = \frac{7}{10}$$

OR $$P(H)+P(ES) = \frac{3}{10}+\frac{4}{10} = \frac{7}{10}$$

Your Turn

A cooler contains the following juice bottles: 3 orange, 5 apple, 3 citrus blend, and 4 grape. What is the probability of reaching in and randomly choosing an apple or grape juice?

Total of $3+5+3+4 = 15$ juice bottles

$$P(A) + P(G) = \frac{5}{15} + \frac{4}{15}$$
$$= \frac{9}{15}$$
$$= \frac{3}{5}$$

Write this down:

Additive Principle (Rule of Sum) for Mutually Exclusive Events

The probability of either of two mutually exclusive events, A or B, is:

$P(A \text{ or } B) = P(A) + P(B)$

Example 2

Apply the Rule of Sum for Mutually Exclusive Events

A number of actors have starred as James Bond over the years. The table summarizes Rolly's Bond movie collection, tallied by actor.

Actor	Number of Movies
Sean Connery	6
George Lazenby	1
Roger Moore	7
Timothy Dalton	2
Pierce Brosnan	4
Daniel Craig	2

Sometimes Rolly likes to pick a Bond movie to watch at random. What is the probability that Rolly will randomly pick either a Connery, C, or a Dalton, D, film from his shelf?

$$P(C) + P(D)$$
$$= \frac{6}{22} + \frac{2}{22}$$
$$= \frac{8}{22} = \frac{4}{11}$$

Example 2

Apply the Rule of Sum for Mutually Exclusive Events

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Actor	Number of Movies
Sean Connery	6
George Lazenby	1
Roger Moore	7
Timothy Dalton	2
Pierce Brosnan	4
Daniel Craig	2

Your Turn

What is the probability that Rolly will randomly pick either a Brosnan or a Moore movie?

$$P(B) + P(M)$$
$$= \frac{4}{22} + \frac{7}{22}$$
$$= \frac{11}{22} = \frac{1}{2}$$

In some situations, it is possible for different events to occur simultaneously. For example, in a standard deck of cards, let $n(D)$ represent the number of diamonds and $n(F)$ represent the number of face cards.

A♣	2♣	3♣	4♣	5♣	6♣	7♣	8♣	9♣	10♣	J♣	Q♣	K♣
A♦	2♦	3♦	4♦	5♦	6♦	7♦	8♦	9♦	10♦	J♦	Q♦	K♦
A♠	2♠	3♠	4♠	5♠	6♠	7♠	8♠	9♠	10♠	J♠	Q♠	K♠
A♥	2♥	3♥	4♥	5♥	6♥	7♥	8♥	9♥	10♥	J♥	Q♥	K♥

From the diagram, $n(D) = 13$ and $n(F) = 12$. Adding these gives

$$\begin{aligned} n(D) + n(F) &= 13 + 12 \\ &= 25 \end{aligned}$$

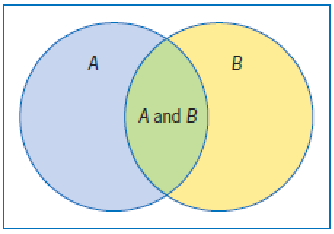
This sum includes three cards that belong to both sets, J♦, Q♦, and K♦, which have been counted twice. This represents an example of **non-mutually exclusive events**.

A♣	2♣	3♣	4♣	5♣	6♣	7♣	8♣	9♣	10♣	J♣	Q♣	K♣
A♦	2♦	3♦	4♦	5♦	6♦	7♦	8♦	9♦	10♦	J♦	Q♦	K♦
A♠	2♠	3♠	4♠	5♠	6♠	7♠	8♠	9♠	10♠	J♠	Q♠	K♠
A♥	2♥	3♥	4♥	5♥	6♥	7♥	8♥	9♥	10♥	J♥	Q♥	K♥

To obtain the correct number of diamonds and face cards, subtract the three cards that were counted twice from the sum found above:

$\boxed{25 - 3 = 22}$

This counting strategy is known as the principle of inclusion and exclusion.



We must subtract the common elements that are part of A and B

Principle of Inclusion and Exclusion

If A and B are non-mutually exclusive events, then the total number of favourable outcomes is:

$n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$

In a similar way...

To calculate the probability of two non-mutually exclusive events, A and B, add the probability of each event and subtract the probability of both events occurring simultaneously.

Probability of Non-Mutually Exclusive Events

The probability of either of two non-mutually exclusive events, A or B, is:

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Example 3

Principle of Inclusion and Exclusion

In a room with 30 students, 10 play basketball and 15 play volleyball. If 7 students play both sports, what is the probability that a student chosen at random plays basketball or volleyball?

$$P(B \text{ or } V) = P(B) + P(V) - P(BV)$$

$$= \frac{10}{30} + \frac{15}{30} - \frac{7}{30}$$

$$= \frac{18}{30}$$

$$= \frac{3}{5}$$

Your Turn

Miranda is part of a gift exchange with 24 other family members and friends. Of the 24 family members, 10 like to ski, 12 like to cycle, and 6 like to ski and cycle. If Miranda randomly draws a name, what is the probability that she will pick someone who likes to ski or cycle?

$$P(S \text{ or } C) = P(S) + P(C) - P(SC)$$

$$= \frac{10}{24} + \frac{12}{24} - \frac{6}{24}$$

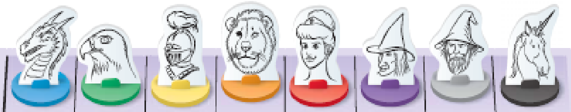
$$= \frac{16}{24}$$

$$= \frac{2}{3}$$

Example 4

Probability of Non-Mutually Exclusive Events

The playing tokens and their characteristics for a role-playing game are shown below.



	Dragon	Hawk	Knight	Lion	Princess	Witch	Wizard	Unicorn
Human			✓		✓	✓	✓	
Animal	✓	✓		✓				✓
Supernatural	✓					✓	✓	✓
Can fly	✓	✓				✓		
Can cast spells						✓	✓	

If Jozo is randomly assigned a playing token, what is the probability that it will be either an animal or a supernatural creature?

$$P(A \text{ or } SN) = P(A) + P(SN) - P(A\,SN)$$
$$= \frac{4}{8} + \frac{4}{8} - \frac{2}{8}$$
$$= \frac{6}{8} = \frac{3}{4}$$

Your Turn

What is the probability that Jozo will randomly pick a flying creature or one that can cast spells?

$$P(F \text{ or } CS) = P(F) + P(CS) - P(F\,CS)$$
$$= \frac{3}{8} + \frac{2}{8} - \frac{1}{8}$$
$$= \frac{4}{8}$$
$$= \frac{1}{2}$$