MTH1W Grade 9 Mathematics

1.3 Powers and Scientific Notation

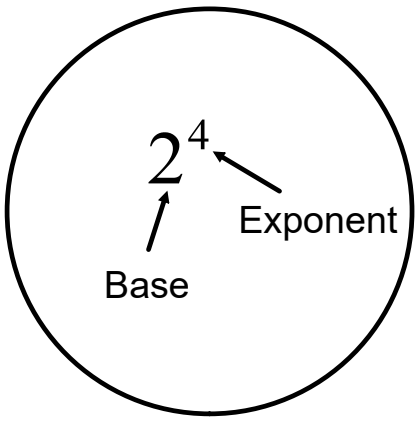
Goal(s) ***- Analyze and understand the sign of an exponent.***

- Evaluate powers with exponents having different signs.

- Express large and small numbers in scientific notation.

Page 22 #s 2, 4abhik, 5bc, 6aceg, 7bdfh, 12acf, 15abhi, 17cd, 20, 23ac, 24bc

Recall that a **power** is a product of identical factors and consists of two parts: a ***base*** and an ***exponent***.



Power

repeated multiplication

$2^4 = 2 \times 2 \times 2 \times 2$

The base is the ***identical factor***, and the exponent tells how many ***factors*** there are.

$$2^4 = 2 \times 2 \times 2 \times 2$$

exponential form expanded form

This can be evaluated (if asked)

$\Rightarrow = 16$

For each power: **identify the base**, **identify the exponent**, and **then evaluate**.

	Base	Exponent
$4^3 =$ 64	3	4
$(-3)^2 =$ 9	-3	2
$2^{-3} =$ 0.125 $\left(\frac{1}{8}\right)$	2	-3
$\left(\frac{3}{4}\right)^4 = \frac{81}{256}$	$\frac{3}{4}$	4

$\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$ multiply tops multiply bottoms

Complete the pattern:

$2^4 = 16$

$2^3 =$ 8 ÷2

$2^2 =$ 4 ÷2

$2^1 =$ 2 ÷2

Describe what is happening each time to the...

- exponents.
- products

exponents → decreasing by 1

products → divided by 2 THE BASE

Complete the pattern:

$2^4 = 16$

$2^3 = 8$

$2^2 = 4$

$2^1 = 2$

Without using your calculator, what do you think the following products should be?

$2^0 =$ 1

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{4}$

$2^{-3} = \frac{1}{8}$

$2^{-4} = \frac{1}{16}$

$2^{-5} = \frac{1}{32}$

$3^3 = 27$

$5^4 = 625$

$7^3 = 343$

$3^{-3} = \frac{1}{27}$

$5^{-4} = \frac{1}{625}$

$7^{-3} = \frac{1}{343}$

Complete the pattern:

$2^4 = 16$

$2^3 = 8$

$2^2 = 4$

$2^1 = 2$

$2^0 = 1$

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{4}$

$2^{-3} = \frac{1}{8}$

$2^{-4} = \frac{1}{16}$

$2^{-5} = \frac{1}{32}$

What rule can we define when asked to evaluate a term with a negative exponent?

Evaluate with a positive exponent

Turn it into a fraction with a numerator of 1.

Evaluating Negative Exponents

When a **base** is raised to a **negative** exponent, it is equal to its **reciprocal** raised to the same, **positive** exponent.

$$4^{-3} = \left(\frac{1}{4}\right)^3 = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$$

1 ÷ by the value

Integers $\longrightarrow \frac{1}{\text{integer}}$

Fractions $\longrightarrow$ flip them

Eg $\frac{2}{3} \longrightarrow \frac{3}{2}$

Express with positive exponents. Then evaluate each power.

$5^{-2} = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$

$\left(\frac{1}{2}\right)^{-6} = (2)^6 = 64$

$(-6)^{-3} = \left(\frac{-1}{6}\right)^3 = \frac{-1}{216}$

$\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$

To write with positive exponents, find the reciprocal of the base.

Evaluating Zero Exponents

When a **base** is raised to a **zero** exponent, it is equal to *1*.

$4^0 = 1$ $\left(\frac{1}{4}\right)^0 = 1$ $(-27)^0 = 1$

Any non-zero base to the power of zero equals 1

$\Rightarrow 1{,}834{,}679^0 = 1$

Scientific Notation

Scientific notation is a **compact way** to write *very large* or very *small numbers*.

To write a number using scientific notation, *place the decimal to the right of the first non-zero digit* and *multiply by a power of 10.*

$1\,456\,000\,000\,000 = 1.456 \times 10^{12}$

first non-zero digit

decimal moves 12 places

$0.000\,001\,23 = 1.23 \times 10^{-6}$

decimal

first

Rewrite each amount in scientific notation.

The distance from Earth to the moon is $384\,000\,000$ meters.

$384000000 \Rightarrow 3.84 \times 10^{8}$ m

8 places

The approximate weight of an ant is $0.000\,010$ kilograms.

$0.000010 \Rightarrow 1.0 \times 10^{-5}$ kg

5 places

The population of Canada in 2021 is 38 million.

$38\,000\,000 \Rightarrow 3.8 \times 10^{7}$ people

7 places

Rewrite in standard notation.

4.327 x 10^{-9} mL

0.000000004327 mL

9 places

$1.89 x 10^{7}

$18,900,000

7 places