Comparing Experimental and Theoretical Probabilities

Lesson objectives

- I can recognise the difference between experimental and theoretical probability

1.1

Lesson objectives | Teachers' notes | Lesson notes

MHR Page 32 #s 1, 2 & 5

Warm Up

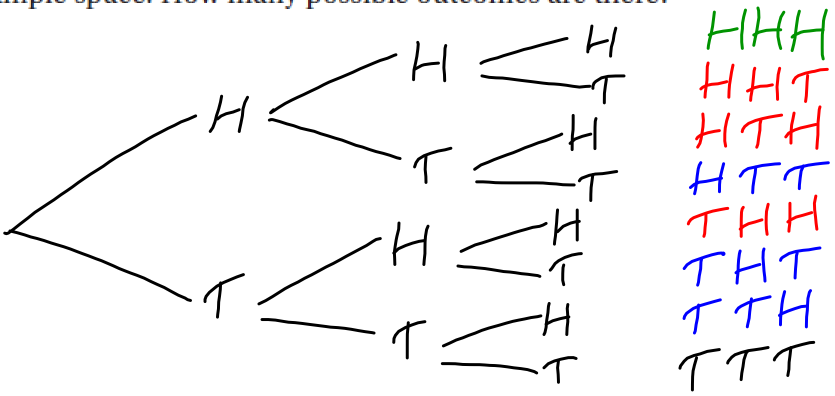
Have you ever wondered what it would be like to fly in space? Not many people get the opportunity to do that, but technology can be used to simulate the real experience. In fact, astronauts spend far more time preparing in a flight simulator than they actually do in space. What other types of simulators have you heard about?

Discuss with the person next to you and be prepared to share your answers.

We are going to use Excel to run a simulation to investigate the number of heads that occur when flipping 3 coins.

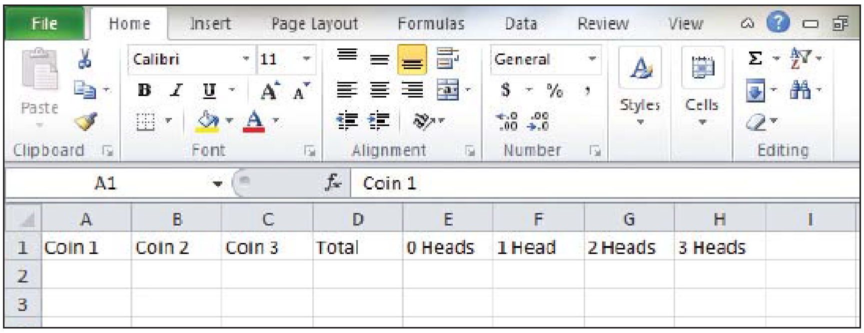
But first, let's calculate what we think the answers should be.

1. Draw a tree diagram to show all possible outcomes. Examine the sample space. How many possible outcomes are there?



2. What is the theoretical probability that there will be:
   a) exactly three heads → $\frac{1}{8}$
   b) exactly two heads → $\frac{3}{8}$
   c) exactly one head → $\frac{3}{8}$
   d) no heads → $\frac{1}{8}$

a) Open a spreadsheet and label eight columns as shown.



	A	B	C	D	E	F	G	H	I
1	Coin 1	Coin 2	Coin 3	Total	0 Heads	1 Head	2 Heads	3 Heads	
2									
3									

b) Program the spreadsheet to simulate three coin tosses:
- Type the following in cell A2: =RANDBETWEEN(0,1)
- Copy the contents of cell A2 into cells B2 and C2.

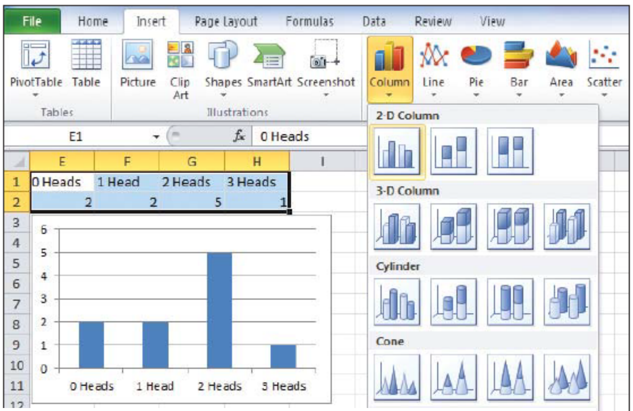
c) Program the spreadsheet to calculate the total number of heads for a given trial:
- Type the following in cell D2: =SUM(A2:C2)

d) Carry out 10 trials:
- Highlight cells A2 to D2.
- Drag the small square in the bottom right corner of D2 down to cell D11.

e) Program the spreadsheet to count the number of heads in each trial. For example, type "=COUNTIF(D:D,0)" into E2 to count how many trials included 0 heads. Use this command for counting 0, 1, 2, or 3 heads.
- Type the following in cell E2: =COUNTIF(D:D,0)
- Type the following in cell F2: =COUNTIF(D:D,1)
- Type the following in cell G2: =COUNTIF(D:D,2)
- Type the following in cell H2: =COUNTIF(D:D,3)

What do these commands do?

f) Create a bar graph to represent the frequency of each outcome:
- Highlight cells E1 to H2.
- Choose the **Insert** tab.
- From the **Column** menu, choose **Clustered Column**.



g) Carry out 100 trials:
- Highlight cells A11 to D11.
- Click and drag the bottom corner of cell D11 to cell D101.

How does the graph compare to the theoretical prediction?

h) Carry out 1000 trials. Describe how the bar graph changes.

4. Reflect Explain what happens to the statistical probabilities of this experiment as the total number of trials increases.

The more trials completed, the closer the experimental probability is to the theoretical probability.

5. Extend Your Understanding Suppose this experiment were modified to include five coins, instead of three.

a) Do you think it would take more, fewer, or about the same number of trials for the statistical probabilities of the outcomes to match the theoretical probabilities? Explain your reasoning.

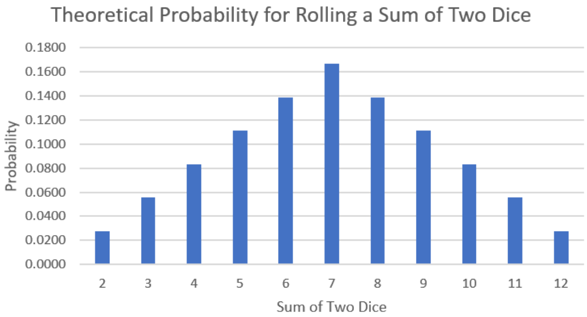
b) Design and carry out this experiment and write a brief report of your findings.

a) More. For 3 coins there are 8 outcomes so you need a minimum of 8 flips to have the probabilities agree. For 5 coins there are 32 outcomes so you need a minimum of 32 flips to have them agree.

The table below illustrates all possible outcomes for the sum of the dice when two standard dice are thrown.

	Outcome	Die 1 ⚀	⚁	⚂	⚃	⚄	⚅
Die 2	⚀	2	3	4	5	6	7
	⚁	3	4	5	6	7	8
	⚂	4	5	6	7	8	9
	⚃	5	6	7	8	9	10
	⚄	6	7	8	9	10	11
	⚅	7	8	9	10	11	12

1. What is the theoretical probability of rolling each sum?
2. Sketch a bar graph showing the theoretical probability of rolling each sum.



$P(2) = \frac{1}{36}$

$P(3) = \frac{2}{36} = \frac{1}{18}$

$P(4) = \frac{3}{36} = \frac{1}{12}$

$P(5) = \frac{4}{36} = \frac{1}{9}$

$P(6) = \frac{5}{36}$

$P(7) = \frac{6}{36} = \frac{1}{6}$

$P(8) = \frac{5}{36}$

$P(9) = \frac{4}{36} = \frac{1}{9}$

$P(10) = \frac{3}{36} = \frac{1}{12}$

$P(11) = \frac{2}{36} = \frac{1}{18}$

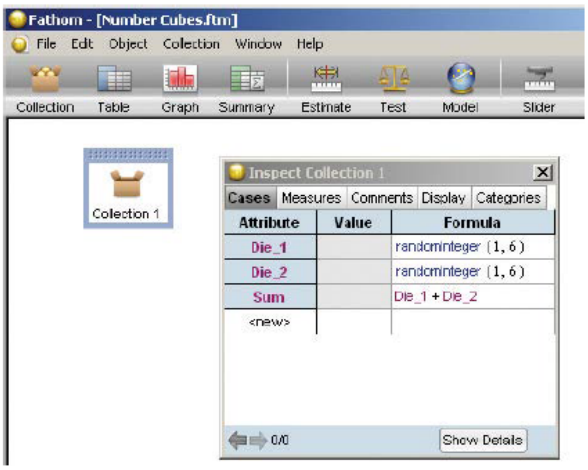
$P(12) = \frac{1}{36}$

We will now use Fathom to run a simulation of rolling two dice.

a) Open Fathom™ and create a new collection by dragging the **Collection Box** into the workspace.

b) Create the dice simulation:
- Double click on **Collection 1** to open the **Inspector**.
- In the Attribute column, enter “Die_1”, “Die_2”, and “Sum” as shown.
- Double click under **Formula** and type the following for each die and then choose **OK**:
  randomInteger(1,6)
- Enter the formula for **SUM** as follows:
  – Click on **Attributes**.
  – Double click on **Die_1**.
  – Click on the **+** key.
  – Double click on **Die_2** and choose **OK**.

What do these commands do?



c) Simulate 10 trials:
- Right click on **Collection 1**.
- From the Collection menu, choose **New Cases**.
- Type 10 and choose **OK**.

Use the left and right arrows in the **Inspector** to view the outcomes.

d) Construct a bar graph to illustrate the outcomes:
- Click and drag a **Graph** into the workspace.
- Click on the **Sum** attribute in the **Inspector** and drag it onto the horizontal axis of the graph.
- Click on **Dot Plot** and change the graph to **Histogram**.

e) Conduct several trials. From the **Collection** menu, choose **New Cases**. Enter 90 and click **OK** to generate a total of 100 cases. Describe how the shape of the graph has changed.

f) Add new cases, 100 at time, and describe how the graph changes Keep adding cases until the statistical probabilities look very similar to the theoretical probabilities. How many cases did you have to use?

4. **Reflect** Explain what happens to the statistical probabilities of this experiment as the total number of trials increases.

5. **Reflect** Approximately how many trials did it take before the statistical probabilities closely agreed with the theoretical probabilities? Why do you think this is so?

6. **Extend Your Understanding** How would the outcomes of this experiment change if you used three dice instead of two? Design and carry out an experiment to find out. Describe your findings.