

Solutions

1. Yuri is playing a card game. He will lose if he draws a face card (J, Q, or K) from a full deck of standard playing cards. What is the theoretical probability that Yuri will win his first draw?

A 6% B 9%
C 23% D 77%

$$P(\text{Face}) = \frac{12}{52}$$

$$= 23\%$$

$$P(\text{win}) = 1 - P(\text{Face})$$

$$= 1 - 23\%$$

$$= 77\% \Rightarrow D$$

2. A weather forecast predicts a 33% chance of rain tomorrow. What are the odds in favour of it raining tomorrow?

A 1:2 B 1:3
C 2:1 D 3:1

$$33\% \text{ rain} \Rightarrow 67\% \text{ not rain}$$

Odds in favour

$$33:67$$

$$\approx 1:2 \Rightarrow A$$

3. Susie is drawing toothpicks with four co-workers to see who will go on a snack run. What are the odds against Susie having to go get the snacks?

A 1:4 B 1:5
C 4:1 D 5:1

5 people

$$P(\text{picked}) = 0.2$$

$$P(\text{not picked}) = 1 - 0.2$$

$$= 0.8$$

odds against being picked

$$0.8 : 0.2$$

$$(\div \text{ by } 0.2) \quad 4:1 \Rightarrow C$$

4. Two standard dice are thrown. Determine the theoretical probability that the sum is

- a) 4
- b) 7
- c) an even number
- d) not a 6
- e) not a perfect square

a) Totals that make 4
(1, 3) (2, 2) (3, 1)

$$P(4) = \frac{3}{36} = \frac{1}{12}$$

b) Totals that make 7
(1, 6) (2, 5) (3, 4) (4, 3) (5, 2) (6, 1)

$$P(7) = \frac{6}{36} = \frac{1}{6}$$

c) $P(2) = \frac{1}{36}$ $P(8) = \frac{5}{36}$
 $P(4) = \frac{3}{36}$ $P(10) = \frac{3}{36}$
 $P(6) = \frac{5}{36}$ $P(12) = \frac{1}{36}$
 $P(\text{even}) = \frac{18}{36} = \frac{1}{2}$

d) $P(\text{not a 6})$
 $= 1 - P(6)$
 $= \frac{36}{36} - \frac{5}{36}$
 $= \frac{31}{36}$

e) $P(1) = \frac{1}{36}$ $P(9) = \frac{4}{36}$
 $P(4) = \frac{3}{36}$
 $P(\text{square}) = 1 - P(\text{not square})$
 $= \frac{36}{36} - \frac{7}{36}$
 $= \frac{29}{36}$

5. A drawer contains 3 black socks, 1 white sock, and 2 grey socks, all of the same style. Two socks are chosen from the drawer at random.

- a) Describe the sample space using set notation.
- b) Use set notation to show the different ways of choosing two socks that are the same colour.
- c) What is the theoretical probability that two randomly chosen socks will be the same colour?
- d) What are the odds in favour of two randomly chosen socks being the same colour?

		Second Pick					
		B1	B2	B3	W1	G1	G2
First Pick	B1	X	B1B2	B1B3	B1W1	B1G1	B1G2
	B2	B2B1	X	B2B3	B2W1	B2G1	B2G2
	B3	B3B1	B3B2	X	B3W1	B3G1	B3G2
	W1	W1B1	W1B2	W1B3	X	W1G1	W1G2
	G1	G1B1	G1B2	G1B3	G1W1	X	G1G2
	G2	G2B1	G2B2	G2B3	G2W1	G2G1	X

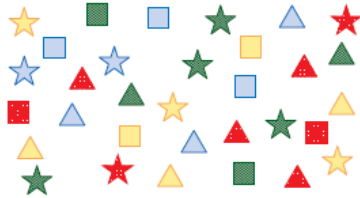
a) $S = \{ B_1B_2, B_1B_3, B_1W_1, B_1G_1, B_1G_2, B_2B_1, B_2B_3, B_2W_1, B_2G_1, B_2G_2, B_3B_1, B_3B_2, B_3W_1, B_3G_1, B_3G_2, W_1B_1, W_1B_2, W_1B_3, W_1G_1, W_1G_2, G_1B_1, G_1B_2, G_1B_3, G_1W_1, G_1G_2, G_2B_1, G_2B_2, G_2B_3, G_2W_1, G_2G_1, G_2G_2 \}$

b) $C = \{ B_1B_2, B_1B_3, B_2B_1, B_2B_3, B_3B_1, B_3B_2, G_1G_2, G_2G_1 \}$

c) $P(\text{same colour}) = \frac{8}{30} = \frac{4}{15}$

d) Odds in favour of same colour
 8 outcomes the same colour
 22 outcomes are not the same colour
 $\Rightarrow 8 : 22$
 (\div by 2) $4 : 11$

6. In a lab study of learned behaviours, monkeys are taught to reach into a box and randomly choose a shape from the ones shown. If the monkey chooses any shape that is not red, he gets a reward. If he chooses a red shape, he gets nothing. By monitoring the monkey's behaviour as several trials are carried out, scientists can see whether there is any evidence that the monkey is able to recognize red.



- a) Determine the theoretical probability that the monkey will be rewarded on any given trial, assuming that he randomly chooses a shape.
- b) Determine the theoretical probability that the monkey in this example does not randomly choose a star.

32 shapes, 8 are red

$$\begin{aligned} \text{a) } P(\text{not red}) &= 1 - P(\text{red}) \\ &= \frac{32}{32} - \frac{8}{32} \\ &= \frac{24}{32} = \frac{3}{4} \end{aligned}$$

11 stars

$$\begin{aligned} \text{b) } P(\text{not star}) &= 1 - P(\text{star}) \\ &= \frac{32}{32} - \frac{11}{32} \\ &= \frac{21}{32} \end{aligned}$$

7. **Communication** Refer to #6. Suppose that after several trials, the experiment is modified to allow the monkey to look into the box while choosing a shape. The table shows the results.

Colour Chosen	Count
Red	2
Not red	58

- a) Determine the experimental probability that the monkey will choose a red shape.
- b) Determine the experimental probability of the complementary event.
- c) What might this suggest to the science researchers? Explain your reasoning.

$$\text{a) } P(\text{red}) = \frac{2}{60} = \frac{1}{30}$$

$$\begin{aligned} \text{b) } P(\text{not red}) &= 1 - P(\text{red}) \\ &= \frac{30}{30} - \frac{1}{30} \\ &= \frac{29}{30} \end{aligned}$$

c) The monkey is choosing "not red" significantly more often compared to when randomly selecting a shape. \Rightarrow monkey recognises red!

8. Emily estimates that the odds against Paulo asking her to the prom are 4:1.

- What type of probability is Emily applying?
- What is the probability that Paulo will ask Emily to the prom, based on her estimate?

a) Subjective probability

b) Odds are against
 $\Rightarrow 4, \text{no} : 1, \text{yes}$
 $P(\text{not ask}) = \frac{4}{5}$
 $P(\text{does ask}) = \frac{1}{5}$

9. Chelsea is trying out for her school play.

Using subjective probability, she estimates that she has an 80% chance of getting a part and a 25% probability of landing a lead role.

- What are the odds in favour of Chelsea getting a part in the play?
- What are the odds against her landing a lead role?

a) 80% yes, 20% no
 Odds in favour

$$80 : 20$$

$$(\div \text{ by } 20) \quad 4 : 1$$

b) 25% lead, 75% not lead
 Odds against

$$75 : 25$$

$$(\div \text{ by } 25) \quad 3 : 1$$

10. Kwon is answering four true or false questions on a quiz. Assume that he randomly guesses each answer.

a) Draw a tree diagram to illustrate all possible outcomes.

b) What is the theoretical probability that he gets

- all four correct?
- exactly three correct?
- fewer than two correct?
- not all incorrect?

Let C = correct
and I = incorrect

$P(\text{all 4 correct}) = \frac{1}{16}$

$P(\text{exactly 3 correct}) = \frac{4}{16} = \frac{1}{4}$

$P(\text{fewer than 2 correct}) = P(0 \text{ correct}) + P(1 \text{ correct}) = \frac{1}{16} + \frac{4}{16} = \frac{5}{16}$

$P(\text{not all incorrect}) = 1 - P(\text{all incorrect}) = \frac{16}{16} - \frac{1}{16} = \frac{15}{16}$