Theoretical Probability

Lesson objectives

- I can calculate theoretical probability

1.1

Lesson objectives | Teachers' notes | Lesson notes

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Warm Up

Board games usually involve a combination of strategy and luck. Some board games use a pair of standard dice.

- How many possible outcomes are there when rolling a pair of standard dice?
- Suppose you are playing a game involving the sum of two dice. Do you think all sums are equally likely?
- Are there different outcomes that can produce a sum of 2?
- What about a sum of 7?

Discuss with the person next to you and be prepared to share your answers.

Definitions

Theoretical probability
- Probability based on analysis of all possible outcomes
- Also called classical probability

Sample Space
- Collection of all possible outcomes
- Sometimes called a sample set

Event
- Set of outcomes that have a common characteristic

Complement
- Set of outcomes not included in an event

Odds in Favour
- Ratio of the probability that an event will happen to the probability that it will not

Odds Against
- Ratio of the probability that an event will not happen to the probability that it will

The **theoretical probability** of an outcome is one based on analyzing all possible outcomes. Unlike experimental probability, no experiment is carried out. All possible outcomes combined make up the **sample space**. It is often useful to combine different outcomes that have something in common. An **event** occurs when any of these similar outcomes occur. For example, the dice pictured above show a sum of 7. But this is not the only outcome that can result in a sum of 7. What are some others?

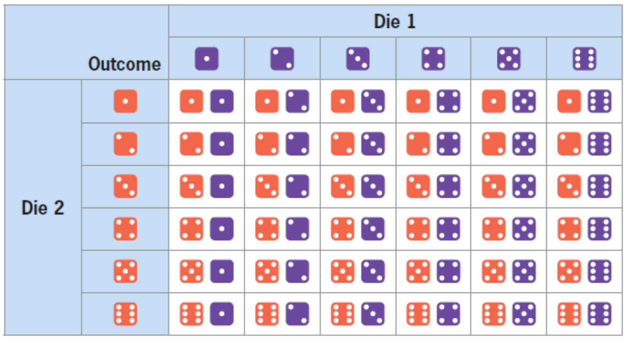
If all outcomes are equally likely, then the theoretical probability of an event, A, is a measure of the ratio of the number of ways it can occur compared to the entire sample space. You can express this probability as a fraction, decimal, or percent.

Theoretical Probability

$$P(A) = \frac{n(A)}{n(S)}$$

where $P(A)$ is the probability that event A can occur, $n(A)$ is the number of ways it can occur, and $n(S)$ is the total number of possible outcomes in the sample space.

This table illustrates the possible outcomes when two standard dice are thrown.



a) Which sum or sums has the greatest theoretical probability? What is the value of this probability?

b) Which sum or sums has the lowest probability? What is the value of this probability?

What is the probability of rolling a 9 or greater?

a) Sum of 7 → $P(7) = \frac{6}{36} = \frac{1}{6}$

b) Of those that are possible...
$P(2)$ and $P(12) = \frac{1}{36}$

c) $P(9 \text{ or greater}) = \frac{10}{36}$ ← 10 ways to score 9 or greater
$= \frac{5}{18}$

Extend Your Understanding Suppose 8-sided dice were used instead, numbered 1 through 8. Would the theoretical probability of rolling each of the following sums increase, decrease, or stay the same? Explain your answers.

a) 2

b) 9

c) doubles

a) decrease $\frac{1}{64}$ vs $\frac{1}{36}$

b) increase $\frac{8}{64}$ vs $\frac{4}{36}$

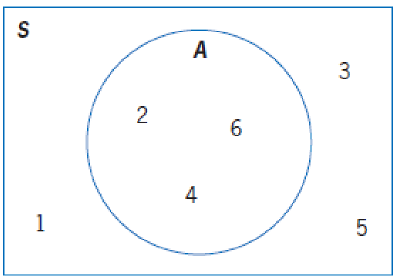
c) decrease $\frac{8}{64}$ vs $\frac{6}{36}$

Instead of using a sample space diagram you can also use a diagram or diagram to help You can also use notation (similar to domain and range).

For example trying to find the probability of rolling an even number on a die...

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{2, 4, 6\}$$



Example 1

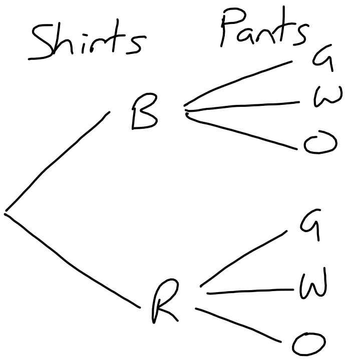
Calculate Theoretical Probability

Fiona has two shirts and three pairs of pants that she can wear to her co-op placement.

a) Suppose Fiona randomly picks a shirt and a pair of pants. Identify the sample space for this probability experiment.

b) Fiona does not like to wear blue and green together, nor does she like orange and red together. What is the theoretical probability that she will randomly pick a shirt and pants combination that she does not like?

c) Suppose Fiona wants to decrease the probability of drawing a combination she does not like. If Fiona buys another pair of pants, which colour should she choose—green, white, or orange—and why?



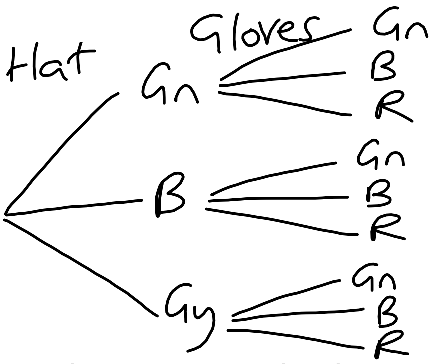
a) Sample space is $S = \{BG, BW, BO, RG, RW, RO\}$

b) Don't want BG or RO.
$\Rightarrow \frac{2}{6} = \frac{1}{3}$

c) Doesn't like BG or RO, so don't buy another pair of green or orange pants $\Rightarrow$ buy white. If we add this to the tree diagram we then have $P(BG \text{ or } RO) = \frac{2}{8} = \frac{1}{4}$ which is less than $\frac{1}{3}$.

Your Turn

Lee has a green hat, a black hat, and a grey hat. She also has green gloves, black gloves, and red gloves. In a hurry, Lee randomly grabs a hat and gloves. Determine the theoretical probability that the hat and gloves are the same colour.



9 possible outcomes

GnGn and BB give the required outcome

$\Rightarrow P(\text{same}) = \frac{2}{9}$

Sometimes you need to know the probability that one event happens compared to all others. If one event is A then event A' (A prime) is all the possible outcomes not in A. This is also known as the **complement** of A. Because the sum of all probabilities in a sample space must equal one, there is a useful relationship between P(A) and P(A').

$P(A) + P(A') = 1$

This relationship can be rearranged into two other useful forms.

$P(A') = 1 - P(A)$ or $P(A) = 1 - P(A')$

Example 2

Probability of a Complement

Battleship is a game in which two opponents use a coordinate grid to try to guess the location of each other's ships.

Examine the board shown. Suppose a location is guessed at random.

a) What is the theoretical probability of hitting a ship on the first guess?

b) What is the probability of a miss on the first guess?

Grid is $10 \times 10 = 100$

Ships are 2, 3, 3, 4, 5 ⇒ Total of 17

a) $P(\text{Hit}) = \frac{17}{100}$

b) $P(\text{Miss}) = 1 - P(\text{Hit})$
$= 1 - \frac{17}{100}$
$= \frac{83}{100}$

Your Turn

A box contains 3 bran, 4 banana, 5 blueberry, and 3 carrot muffins. What is the theoretical probability that you will not randomly choose a blueberry muffin?

Total of $3 + 4 + 5 + 3 = 15$ muffins

$P(\text{Blueberry}) = \frac{5}{15} = \frac{1}{3}$

⇒ $P(\text{Not blueberry}) = 1 - \frac{1}{3}$
$= \frac{2}{3}$

One application of probability, often used in sports, is odds. Odds can be expressed as the odds in favour of an event occurring or the odds against an event occurring. In sports it is actually more common to give the odds against something happening.

Odds

The odds in favour of $A = P(A) : P(A')$

The odds against $A = P(A') : P(A)$

Sports analysts often make predictions about a player's or team's chances for winning a tournament or championship. Often these predictions involve subjective probability based on the analyst's understanding of the player's or team's relative skill level.

Sat 15/06 01:00

2019 STANLEY CUP

NHL

Toronto Maple Leafs	8/1	San Jose Sharks	20/1	Florida Panthers	40/1
Tampa Bay Lightning	9/1	Calgary Flames	25/1	New Jersey Devils	40/1
Boston Bruins	10/1	Chicago Blackhawks	25/1	Carolina Hurricanes	50/1
Nashville Predators	10/1	Columbus Blue Jackets	25/1	Montreal Canadiens	50/1
Pittsburgh Penguins	10/1	Dallas Stars	25/1	New York Islanders	66/1
Vegas Golden Knights	11/1	Los Angeles Kings	25/1	New York Rangers	66/1
Winnipeg Jets	11/1	Minnesota Wild	25/1	Arizona Coyotes	80/1
Washington Capitals	12/1	Philadelphia Flyers	28/1	Buffalo Sabres	80/1
Edmonton Oilers	16/1	St. Louis Blues	33/1	Detroit Red Wings	80/1
Anaheim Ducks	20/1	Colorado Avalanche	40/1	Vancouver Canucks	80/1
				Ottawa Senators	100/1

Example 3

Odds of an Event

a) A hockey analyst gives the Canadian women's hockey team a 75% probability of winning the gold medal in the next Winter Olympics. Based on this prediction, what are the odds in favour of Canada winning Olympic gold?

b) A local sports journalist estimates that the high school boys' soccer team has a 40% probability of going to the OFSAA championship tournament. What are the odds against the boys making OFSAA?

a) $P(win) = 0.75$
$P(not\ win) = 1 - 0.75$
$= 0.25$
$\Rightarrow$ $0.75 : 0.25$
$3 : 1$
in favour of winning

b) $P(A) = 40\% = 0.4$
$P(A') = 1 - P(A)$
$= 1 - 0.4$
$= 0.6$
$\Rightarrow$ $0.6 : 0.4$
$3 : 2$
against getting to OFSAA

Your Turn

a) A sports commentator claims that the Toronto Raptors have a 60% probability of making the playoffs. Based on this estimate, what are the odds in favour of the Raptors making the playoffs?

b) It is estimated that a golfer has a 20% chance of winning a tournament. What are the odds against this golfer winning the tournament?

a) $P(A) = 60\% = 0.6$
$P(A') = 1 - P(A)$
$= 1 - 0.6$
$= 0.4$
favour : against
$\Rightarrow$ $0.6 : 0.4$
$3 : 2$
in favour of making the playoffs

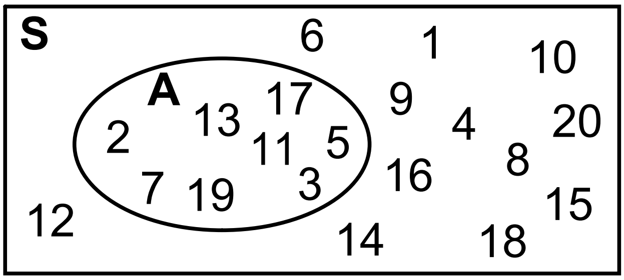
b) $P(A) = 20\% = 0.2$
$P(A') = 1 - P(A)$
$= 1 - 0.2$
$= 0.8$
against : favour
$\Rightarrow$ $0.8 : 0.2$
$4 : 1$
against winning

R1. a) Describe how the terms outcome, event, and sample space are related in terms of theoretical probability. Use a diagram, mind map, or other visual organizer to support your explanation.

b) Create an example that illustrates your answer to a).

a) The set of all possible outcomes is the sample space. An event is a set of outcomes in the sample space that have a common characteristic. Then, the probability of an event A happening is the number of outcomes in that subset divided by the total number of outcomes in the sample space.

b)



S = {1 - 20} A = {Prime < 20}

$$P(A) = \frac{n(A)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

R2. a) Explain how an event is related to its complement.

b) Create an example of an event and its complement, and determine their theoretical probabilities.

a) The complement of an event is the set of possible outcomes not included in an event.

b) The weather forecast is predicting rain this afternoon. It says there is a 70% chance. What is the probability of it not raining this afternoon?

$$P(R') = 1 - P(R)$$
$$P(R') = 1 - 0.70$$
$$P(R') = 0.30$$
$$P(R') = 30\%$$

R3. a) What does odds in favour of an event mean?

b) What does odds against an event mean?

c) How are these concepts similar? How are they different?

a) Odds in favour of an event is a ratio of the probability that an event will happen to the probability that it will not happen.

b) Odds against an event is a ratio of the probability that an event will not happen to the probability that it will happen.

c) They are similar in that they use the same probabilities. They are different in that the order is important.

Homework

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